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A hybrid collocation method based on combining the third kind Chebyshev polynomials and block-pulse functions for solving higher-order initial value problems

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Abstract

The purpose of this paper is to propose a new collocation method for solving high order linear and nonlinear differential equations on a very large interval as well as solving the differential equation problem on a small interval. The new collocation method is based on a hybrid method combining the third kind Chebyshev polynomials and Block-Pulse functions. In the proposed method, the large interval of the problems is divided into small sub-intervals and in each sub-interval, collocation method turns the differential equation into a set of algebraic equations. Solving such system makes an approximate solution of the differential equation on each sub-interval. The error of approximate solution has upper-bound of $O(\frac{m^{-r}}{\sqrt{N}})$. It means that, the errors decrease as m and N increase. The proposed method is more accurate than the previous methods. Numerical examples show the capability and efficiency of the presented method compared to existing methods.

Keywords: Block-puls functions; collocation method; Chebyshev polynomials; higher-order initial value problems (HOIVPs); hybrid collocation method.

1. Introduction

Most problems in other sciences such as the control theory, mechanical systems, the beam theory and physical phenomena lead to ordinary differential equations with initial conditions (Allison, 1970; Berstein *et al*., 1963; Fehlberge, 1969). Therefore, it is very important to solve these equations. In this paper, the HOIVPs are overall examined as follows:

$$f(x, y, y', y'', \ldots, y^{(d)}) = 0\ , \quad 0 \leqslant x \leqslant L\ , \qquad (1)$$

$$y^{(l)}(0) = y_0^{(l)} \quad l = 0, 1, \ldots, d-1. \qquad (2)$$

So far, these problems have been solved in different ways (Abdel-Halim, 2004; Awoyemi, 2003; Kayode & Awoyemi, 2010; Awoyemi & Idowu, 2005; Karimi *et al*., 2013; Kayode, 2008; Olabode, 2009; Lakestani & Dehghan, 2010; Tavassoli *et al*., 2012; Vanden *et al*., 2000; Waeleh *et al*., 2011; Waeleh *et al*., 2012; Yüzbaşi, 2011; Yüzbaşi, 2012; Yüzbaşi, 2013; Yüzbaşi & Şahin, 2012). For example, the exponentially Runge-Kutta method (EFRK) was expressed by Vanden *et al*., 2000 to solve these equations. Likewise, Waeleh and coworkers have also presented some methods in this field (Waeleh *et al*., 2011; Waeleh *et al*., 2012). Their last method known as numerical solution of higher order ordinary differential equations by direct block code (2012) was presented to solve high-order ordinary differential equations with the initial conditions (Waeleh *et al*., 2012). Some of these methods do not have the ability to solve the HOIVPs on large intervals (Abdel-Halim, 2004; Awoyemi, 2003; Awoyemi & Idowu, 2005; Kayode, 2008; Kayode & Awoyemi, 2010; Lakestani & Dehghan, 2010; Olabode, 2009). On the other hand, some methods solve this problem on large intervals but thay suffer a high accuracy (Vanden *et al*., 2000; Olaboode, 2009). Recently, a Legendre wavelet spectral collocation method for solving oscillatory initial value problems as a solution to these equations on large intervals, was also considered in Karimi *et al*., 2013.

Hence in this paper, the aim is solving HOIVPs and initial value problems on large interval and compairing the result with other methods. The proposed method is easy to implement and produces very accurate results. Acordingly, Block-Pulse functions are introduced, third Chebyshev polynomials are presented, and a combination of them is defined, respectively. Moreover, in order to solve these equations, the Hybrid collocation method based on the combination function is used. As well as, in section (4) the upper-bound of absolute error of this method is obtained. Hybrid functions are used in some method for solving initial value problems and integrall equations (Hsiao, 2009; Maleknejad *et al.*, 2011; Maleknejad & Tavassoli, 2003; Marzban & Razzaghi, 2003). This method is also used to solve HOIVPs on large domain. Finally, the results of the present study are compared with the previous methods in terms of accuracy.

2. Function and hybrid function

2.1. Block-pulse functions

A set of block pulse functions $b_i(\lambda)$, $i = 1, \ldots, N$ in the interval $[0, T)$ is defined as:

$$b_i(\lambda) = \begin{cases} 1, & \frac{(i-1)T}{N} \leq \lambda < \frac{iT}{N} \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

These functions are disjointed:

$$b_i(\lambda)b_j(\lambda) = \begin{cases} b_i(\lambda), & \text{for } i = j \\ 0, & \text{for } i \neq j. \end{cases}$$

The advantage of the block pulse function technique is their easy operations and satisfactory approximations. These advantages are due to the distinct properties of block pulse functions including disjointness, orthogonality and completeness (Jiang & Schaufelberger, 1991).

2.2. Third kind of Chebyshev polynomials

A Sturm-Liouville problem is defined as: an eigenvalue problem in the interval $(-1, 1)$ of the form:

$$-\frac{d}{dx}(p(x)\frac{du}{dx}) + q(x)u(x) = \lambda w(x)u(x) \tag{4}$$

on $(-1, 1)$ in which boundary conditions are cosidered. Where p, q, w are three given, realvalued functions in the domain problem $(-1, 1)$. For more details one can see Canuto *et al.*, 2006. The third kind of Chebyshev polynomial $v_n(x)$ is a polynomial of degree n in x defined by

$$v_n(x) = \frac{\cos(n + \frac{1}{2})\theta}{\cos \frac{1}{2}\theta} \tag{5}$$

where $x = \cos\theta$. Based on definition (5), the fundamental recurrence relation is obtained:

$$v_n(x) = 2xv_{n-1}(x) - v_{n-2}(x), \quad n = 2, 3, \ldots$$

with initial conditions

$$v_0(x) = 1, \quad v_1(x) = 2x - 1.$$

The polynomials $v_n(x)$ are, in fact, rescaling of two particular Jacobi polynomials $j_n^{(\alpha,\beta)}(x)$ with $\alpha = \frac{-1}{2}, \beta = \frac{1}{2}$. Explicitly,

$$\begin{pmatrix} 2n \\ n \end{pmatrix} v_n(x) = 2^{2n} j_n^{(\frac{-1}{2},\frac{1}{2})}(x). \tag{6}$$

If the range of variable x is interval $[-1, 1]$, then the range of the corresponding variable θ can be taken as $[0, \pi]$. These ranges are traversed in opposite directions, since $x = -1$, corresponds to $x = \pi$ and $x = 1$ corresponds to $\theta = 0$. These polynomials are orthogonal on $[-1, 1]$ such that the weight function is $w(x) = \sqrt{\frac{1+x}{1-x}}$. Which is,

$$\int_{-1}^{1} v_i(x)v_j(x)w(x)dx = \sqrt{\pi}\delta_{ij}$$

where δ_{ij}, is the Kronecker delta function (John *et al.*, 2003).

Theorem 2.1. If v_n is third kind of Chebyshev polynomial, the derivative of this function (for $n \geq 1$) is obtained in terms of this function as follow:

$$v_n'(t) = \begin{cases} \sum_{i=1}^{k}(2k+2i)v_{2i-1}(t) & n = 2k \\ +\sum_{i=0}^{k-1}(2k-2i)v_{2i}(t), & \\ \sum_{i=1}^{k}(2k-2i)v_{2i-1}(t) & n = 2k-1. \\ +\sum_{i=0}^{k-1}(2k+2i)v_{2i}(t), & \end{cases} \tag{7}$$

and $v_0'(t) = 0$.

proof. The proof is clear and hence omitted.

Corollary 2.2. The second derivative for $n > 1$ can be obtained as follows. In fact,

if $n = 2k$,

$$\begin{aligned} v_n''(t) &= \sum_{i=1}^{k}(2k+2i)\left[\sum_{j=1}^{i}(2i-2j)v_{2j-1}(t) + \sum_{j=0}^{i-1}(2i+2j)v_{2j}(t)\right] \\ &+ \sum_{i=1}^{k-1}(2k-2i)\left[\sum_{j=1}^{i}(2i+2j)v_{2j-1}(t) + \sum_{j=0}^{i-1}(2i-2j)v_{2j}(t)\right] \end{aligned} \tag{8}$$

and if $n = 2k-1$,

$$
\begin{aligned}
v''_n(t) &= \sum_{i=1}^{k}(2k-2i)\left[\sum_{j=1}^{i}(2i-2j)v_{2j-1}(t) + \sum_{j=0}^{i-1}(2i+2j)v_{2j}(t)\right] \\
&\quad + \sum_{i=1}^{k-1}(2k+2i)\left[\sum_{j=1}^{i}(2i+2j)v_{2j-1}(t) + \sum_{j=0}^{i-1}(2i-2j)v_{2j}(t)\right] \qquad (9)
\end{aligned}
$$

and for $n = 0, v''_0(t) = 0$.

proof. To obtain this result, derivative of Equation (7) can be used.

Similarly, higher order derivatives of these functions can also be calculated. Further, suppose that $z_i, i = 1, \cdots, m$ be roots of v_m such that:

$$-1 < z_1 < z_2 < \cdots < z_m < 1, \qquad (10)$$

which are called Chebyshev-Gauss points of third kind.

2.3. Hybrid functions and their derivatives

The hybrid chebyshev block-pulse functions are defined as follows:

Definition: For $i = 1, \dots, N$ and $j = 0, \dots, M-1$, the hybrid third kind of Chebyshev and blockpulse functions are defined as:

$$
p_{i,j}(t) = \begin{cases} \sqrt{\frac{2T}{N}}v_j(\frac{2N}{T}t-2i+1), & \frac{(i-1)T}{N} \le t < \frac{iT}{N} \\ 0, & \text{otherwise.} \end{cases} \qquad (11)
$$

The derivative of $p_{i,j}$ is obtained subsequently:

$$
p'_{i,j}(t) = \begin{cases} \sqrt{\frac{2T}{N}}\frac{2N}{T}v'_j(\frac{2N}{T}t-2i+1), & \frac{(i-1)T}{N} \le t < \frac{iT}{N} \\ 0, & \text{otherwise,} \end{cases} \qquad (12)
$$

and

$$
p''_{i,j}(t) = \begin{cases} \sqrt{\frac{2T}{N}}(\frac{2N}{T})^2v''_j(\frac{2N}{T}t-2i+1), & \frac{(i-1)T}{N} \le t < \frac{iT}{N} \\ 0, & \text{otherwise.} \end{cases} \qquad (13)
$$

These derivatives and higher order derivatives of these functions can be written using Equations (7-8) in terms of Chebyshev polynomials. For example, if $j = 2k$, then

$$
p'_{i,j}(t) = \begin{cases} 2\sqrt{\frac{2N}{T}}\left(\sum_{i=1}^{k}(2k+2i)v_{2i-1}(\frac{2N}{T}t-2n+1) + \sum_{i=0}^{k-1}(2k-2i)v_{2i}(\frac{2N}{T}t-2n+1)\right), & \frac{(i-1)T}{N} \le t < \frac{iT}{N} \\ 0, & \text{otherwise,} \end{cases} \qquad (14)
$$

and if $j = 2k-1$, then:

$$
p'_{i,j}(t) = \begin{cases} 2\sqrt{\frac{2N}{T}}\left(\sum_{i=1}^{k}(2k-2i)v_{2i+1}(\frac{2N}{T}t-2n+1) + \sum_{i=0}^{k-1}(2k+2i)v_{2i+2}(\frac{2N}{T}t-2n+1)\right), & \frac{(i-1)T}{N} \le t < \frac{iT}{N} \\ 0, & \text{otherwise.} \end{cases} \qquad (15)
$$

Higher order derivatives of these functions can also be calculated.

3. Hybrid collocation method

In this study, the approximate solution of Equation (1) is obtained by

$$\widehat{y}(x) = \sum_{i=1}^{N}\widehat{y}_i(x), \qquad (16)$$

such that $\widehat{y}_i$ is the approximate solution on the nth subinterval $[\frac{(i-1)T}{N}, \frac{iT}{N}]$, then

$$\widehat{y}_i(x) = \sum_{j=0}^{m-1}p_{i,j}(x)c_{i,j}, \qquad (17)$$

where $c_{i,j}$ are unknown coefficients. The derivative of this function is obtained as:

$$\widehat{y}_i^{(s)}(x) = \sum_{j=0}^{m-1}p_{i,j}^{(s)}(x)c_{i,j}. \qquad (18)$$

The residual function for ith sub-interval with substitution of the approximate solution given in Equation (17) and its derivatives (18) into Equation (1) is defined as:

$$Res(x) = f\left(x, \sum_{j=0}^{m-1} p_{i,j}(x)c_{i,j}, \sum_{j=0}^{m-1} p'_{i,j}(x)c_{i,j}, \cdots, \sum_{j=0}^{m-1} p_{i,j}^{(d)}(x)c_{i,j}\right). \quad (19)$$

The hybrid collocation method is used for Equation (19) on collocation points x_k^i where collocation points are defined as follows:

$$x_k^i = \frac{2N}{T}(z_k - \frac{iT}{N}) + 1, \quad \begin{array}{l} k = 1, 2, \cdots, m-d+1 \\ i = 1, 2, \cdots, N. \end{array}$$

Its worth mentioning that z_k are introduced in (10). Therefore, a system of $m+1$ algebraic equations for unknown cofficient $c_{i,j}$ on ith subinterval is obtained as following equations:

$$\begin{cases} Res(x_k^i) = 0, & k = 1, \cdots, m-d+1 \\ \sum_{j=0}^{m-1} p_{i,j}^{(l)}(\frac{(i-1)T}{N})c_{i,j} = y_i^{(l)}, & l = 0, \cdots, d-1 \end{cases} \quad (20)$$

$$y_i^{(l)} = \begin{cases} y_0^{(l)}, & i = 1 \\ y_{i-1}^{(l)}(\frac{(i-1)T}{N}), & i = 2, 3, \cdots, N. \end{cases} \quad (21)$$

Obviously, for $i = 1$, from initial conditions in Equation (2), $y_i^{(l)}$ are known. Moreover, for $i = 2, 3, \cdots, N$, considering that $y_{i-1}^{(l)}$ has been calculated, $y_i^{(l)}$ is obtained. In order to obtain the unknown coeffitionts, the system of agebraic equations (20) should be solved. The approximate solution of ith sub-interval is obtained by Equation (17). This procedure is repeated in all sub-intervals and consequently the approximate solution of the original initial value problem is obtained by Equation (16).

4. Error analysis

In this section the upper-bound of the approximate function $(\widehat{y}(x))$ of our scheme is achieved as the following theorem.

Theorem 4.1. Suppose that

$$y \in H^r_{(1-t)^{r-1/2}(1+t)^{r+1/2}}(-1, 1) \quad (\text{integer } r \geq 0)$$

and if $\alpha = \gamma = \frac{-1}{2}$ and $\beta = \delta = \frac{1}{2}$, then:

$$\| I_m^{(-1,1)}y - y \| \leq c(\alpha, \beta)m^{-r}\left(\int_{-1}^{1}(1-t)^{r-\frac{1}{2}}(1+t)^{r+\frac{1}{2}}(\frac{d^r y(t)}{dt^r})^2 dt\right)^{\frac{1}{2}}, \quad (22)$$

Where $I_m^{(-1,1)}y = \widehat{y}(x)$, and $c(\alpha, \beta)$ is a constant depending on α and β, $H_\chi^r(A)$ is weighted Sobolev space.

proof. By taking $\alpha = \frac{-1}{2}$ and $\beta = \frac{1}{2}$ in (6) third kind of Chebyshev polynmiyals is achieved from that Jacobi polynomials. On the other hand, if in theorem 4.3. of Guo & Wang, 2004, the theorem (2.2) of Guo, 2000 is used instead of lemma 2.6, the desired result will be achieved.

Then, upper-bound of approximate function of this method will cosist of m^{-r}. Accordingly, on $(0, T)$:

$$\| I_m^{(0,T)} - y \| \leq c(\alpha, \beta)m^{-r}\left(\int_0^T t^{r-\frac{1}{2}}(T-t)^{r+\frac{1}{2}}(\frac{d^r y(t)}{dt^r})^2 dt\right)^{\frac{1}{2}}.$$

In our scheme, interval $(0, T)$ is divided to N sub-interval as follows:

$$[\frac{(i-1)T}{N}, \frac{iT}{N}], \text{ for } i = 1, \cdots, N.$$

Therefore, the upper-bound of approximate solution of i-th sub-intervals as:

$$\| I_m^{(\frac{(i-1)T}{N}, \frac{iT}{N})}y_i - y \| \leq c(\alpha, \beta)m^{-r} \times \left(\int_{\frac{(i-1)T}{N}}^{\frac{iT}{N}}(t - \frac{(i-1)T}{N})^{r-\frac{1}{2}}(\frac{iT}{N} - t)^{r+\frac{1}{2}}(\frac{d^r y(t)}{dt^r})^2 dt\right)^{\frac{1}{2}}. \quad (23)$$

Furthemore, let

$$M_i = \max_{\frac{(i-1)T}{N} \leqslant t \leqslant \frac{iT}{N}}(t - \frac{(i-1)T}{N})^{r-\frac{1}{2}}(\frac{iT}{N} - t)^{r+\frac{1}{2}}(\frac{d^r y(t)}{dt^r})^2.$$

Thus by using the property of definite integral on right hand sight of (23):

$$\| I_m^{(\frac{(i-1)T}{N}, \frac{iT}{N})}y_i - y \| \leq c(\alpha, \beta)m^{-r}M_i\frac{T}{N},$$

consequently, the upper-bound of approximate function in each subinterval is $O(\frac{m^{-r}}{\sqrt{N}})$. Therfore, our method might be more accurate by choosing large mode m and N. Also, the numerical reasults of examples in the next section show that, the errors decrease for any increase in m and N. Finally, the convergence rate of this method is $O(\frac{m^{-r}}{\sqrt{N}})$.

5. Numerical examples

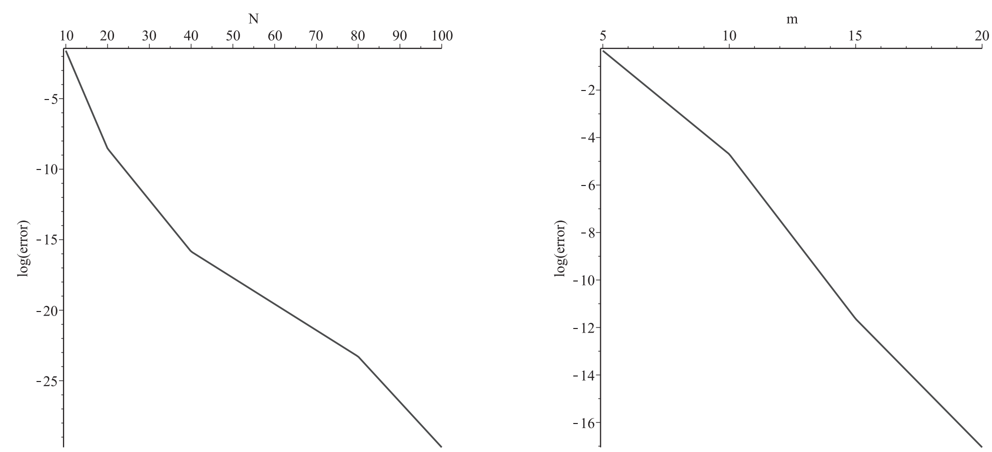
To demonstrate the effectiveness of the proposed method, several examples are presented in this section and the results are compared with five different methods. The examples have been extracted from the latest articles in this field. By comparing the results of our method with other methods, the development of accuracy and efficiency of the method is shown. The numerical comparison of methods using the infinite norms is depicted in the tables. The computer application program ``$Maple18$`` was used to execute the algorithm used in the numerical examples. In examples 1 and 2, the results are compared with the exponentially fitted Runge-Kutta methods (EFRKM) in Vanden *et al*., 2000 and the Legendre wavelets spectral method (LWSM) in Karimi Dizicheh *et al*., 2013.

Example 1. Consider the equation

$$\begin{cases} y'' = -30\sin(30x), & 0 \le x \le 10, \\ y(0) = 0, y'(0) = 1, \end{cases} \tag{24}$$

where the exact solution is $y(x) = \dfrac{\sin(30x)}{30}$.

Table 1 shows the infinite norms of the errors related to our method, LWSM (Karimi *et al*., 2013), and EFRKM (Vanden *et al*., 2000). Clearly our method is more accurate than other methods. Forthermore, the Figure 1 shows that the errors decrease for any increase in m and N.



(a) (b)

Fig. 1. Point-wise absolute errors of Example 1 with $m = 25$ in (a) and $N = 80$ in (b)

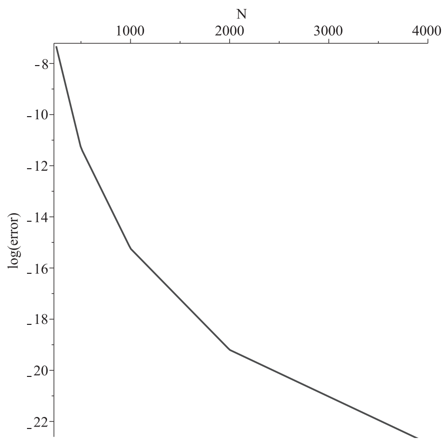
Table1. comparison of Example 1

Method	m	N	2^{k-1}T	Error
The proposed method	26	40	–	$2\ 30 \times 10^{-16}$
	27	40	–	7.60×10^{-19}
	25	80	–	5.20×10^{-24}
	27	80	–	7.00×10^{-27}
	25	80	–	1.38×10^{-31}
LWSM(Karimi *et al*., 2013)	26	–	40	5.69×10^{-14}
	27	–	40	8.24×10^{-16}
	25	–	80	2.11×10^{-21}
	27	–	80	3.84×10^{-24}
	25	–	80	8.32×10^{-29}
EFRKM(Vanden *et al*., 2000)	–	–	–	1.70×10^{-14}

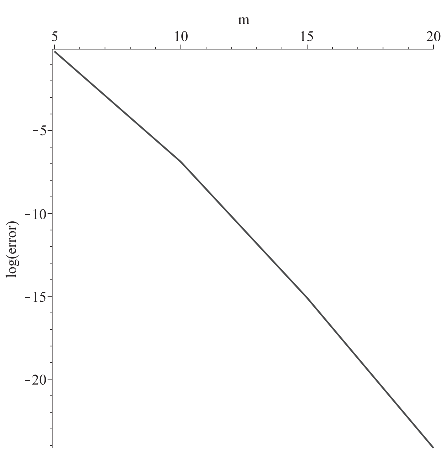
Example 2. Consider the equation

$$\begin{cases} y'' = -y + 0.001\cos(x), \quad 0 \le x \le 1000, \\ y(0) = 1, y'(0) = 0, \end{cases} \tag{25}$$

where the exact solution is $y(x) = \cos(x) + 0.0005x\sin(x)$. Table 2 and Figure 2 show the numerical result to this example which is similar to the result in Example 1.



(a)



(b)

Fig. 2. Point-wise absolute errors of Example 2 with $m = 15$ in (a) and $N = 1000$ in (b)

Table 2. comparison for different methods of Example 2

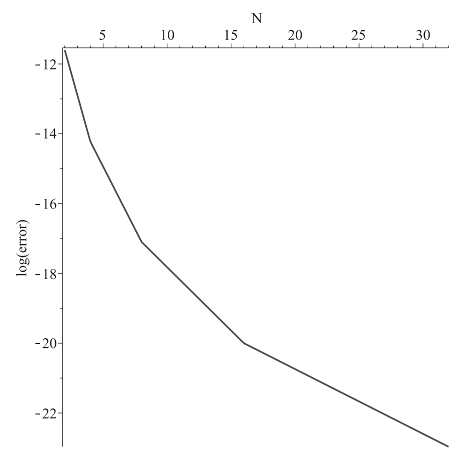
Method	m	N	2^{k-1}T	Error
The proposed method	15	1000	—	8.00×10^{-16}
	15	2000	—	1.00×10^{-19}
	12	4000	—	2.37×10^{-17}
LWSM(Karimi *et al.*, 2013)	12	—	4000	1.08×10^{-16}
EFRKM(Vanden *et al.*, 2000)	—	—	—	2.07×10^{-7}

Example 3. Consider the non-linear HOIVP

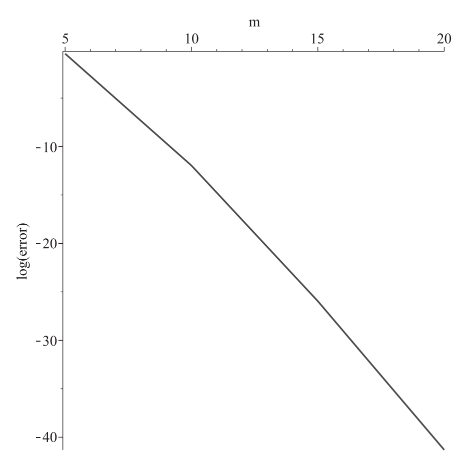
$$\begin{cases} y^{(v)} = 2y'y'' - yy^{iv} - y'y''' - 8x + (x^2 - 2x - 3)e^x, \\ \qquad\qquad\qquad\qquad 0 \le x \le 2, \\ y(0) = 1, y'(0) = 1, y''(0) = 3, \\ y'''(0) = 1, y^{(iv)}(0) = 1, \end{cases} \tag{26}$$

where the exact solution is $y(x) = e^x + x^2$.

Table 3 shows the infinite norms of the errors related to this method, the direct block code method (DBCM) in Waeleh *et al.*, 2012, and the multi derivative collocation method (MCM) in Kayode & Awoyemi, 2010. As we can see, the suggested method is more accurate than DBCM and MCM methods. Furthermore, Figure 3 shows that the errors decrease as m and N increase.



(a)



(b)

Fig. 3. Point-wise absolute errors of Example 3 with $m = 15$ in (a) and $N = 64$ in (b)

Table 3. Numerical comparison for different methods of Example 3

Method	m	N	Error
The proposed method	12	16	6.40×10^{-14}
	15	16	1.00×10^{-20}
	15	64	1.09×10^{-26}
DBCM(Waeleh *et al.*, 2012)	—	—	1.59×10^{-12}
MCM(Kayode & Awoyemi, 2010)	—	—	1.64×10^{-6}

Example 4. Consider the non-linear HOIVP

$$\begin{cases} y^{(iv)} = y'^2 - yy''' - 4x^2 + (1 - 4x + x^2)e^x, \\ \qquad\qquad\qquad\qquad\qquad 0 \le x \le 1, \\ y(0) = 1, y'(0) = 1, y''(0) = 3, y'''(0) = 1, \end{cases} \quad (27)$$

where the exact solution is $y(x) = x^2 + e^x$.

Table 4 shows the infinite norms of the errors related to our method and the efficient zerostable numerical method (EZNM) in Kayode, 2008. It indicates that the method in the present study is more accurate than the EZNM method.

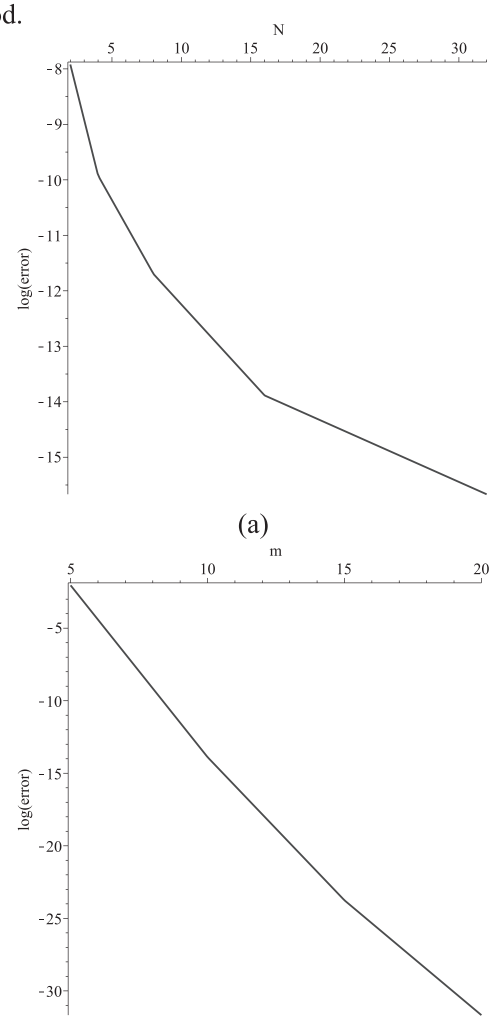


Fig. 4. Point-wise absolute errors of Example 4 with $m = 10$ in (a) and $N = 16$ in (b)

Table 4. Numerical comparison for different methods of Example 4

Method	m	N	Error
The proposed method	8	16	6.80×10^{-10}
	10	16	1.30×10^{-14}
	10	32	2.15×10^{-16}
EZSNM(Kayode, 2008)	—	32	1.59×10^{-7}

Example 5. Consider the non-linear HOIVP

$$\begin{cases} y^{(iv)} = x, \qquad 0 \le x \le 100, \\ y(0) = 1, y'(0) = 1, y''(0) = 3, y'''(0) = 1, \end{cases} \quad (28)$$

where the exact solution is $y(x) = \frac{x^5}{120} + x$.

Table 5 shows the infinite norms of the errors related to this method and a Six-step scheme for the solution of fourth order ordinary differential equations (SSFODEs) in Olabode, 2009. Figure 4 shows that present method is more accurate than that in spite of choosing very small values of m and N, the errors are very small.

Table 5. Numerical comparison for different methods of Example 5

Method	m	N	Error
The proposed method	6	1	3.00×10^{-91}
SSFODEs(Olabode, 2009)	—	—	2.43×10^{-9}

Example 6. Consider the following Riccati differential equation

$$\begin{cases} y' = \frac{-1}{1+t} + y - y^2, \quad 0 \le x \le 1, \\ y(0) = 1, \end{cases} \quad (29)$$

where the exact solution of this example is $y(t) = \frac{1}{1+t}$.

Table 6 shows the infinite norms of the errors related to this method, the cubic b-spline scaling functions (CBSFs) and Chebyshev cardinal functions (CCFs) in Lakestani & Dehghan, 2010. As expected, Table 6 and Figure 5 show the proposed method is more accurate than CBSFs and CCFs methods.

Table 6. Numerical comparison for different methods of Example 6

Method	m	N	Error
The proposed method	15	32	1.84×10^{-31}
	15	64	1.09×10^{-35}
	17	64	1.49×10^{-40}
CBSFs(Lakestani & Dehghan, 2010)	—	10	1.60×10^{-9}
CCFs(Lakestani & Dehghan, 2010)	29	—	1.20×10^{-22}

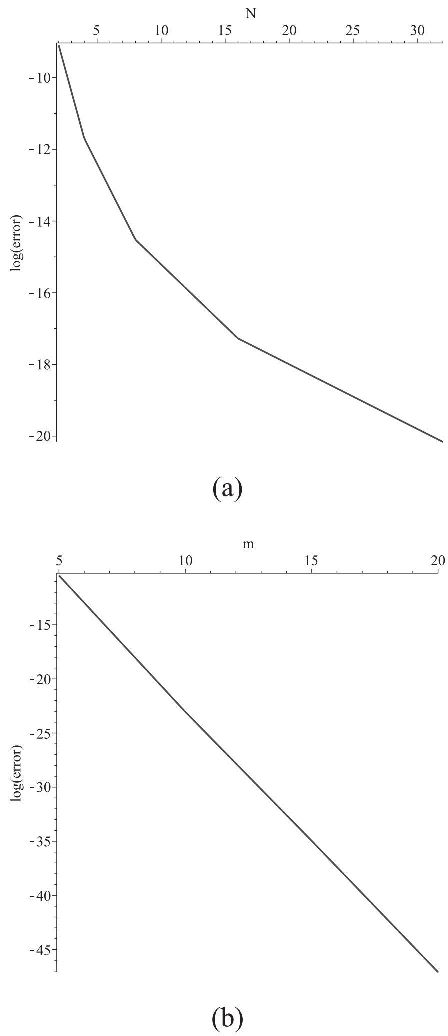


Fig. 5. Point-wise absolute errors of Example 6 with $m = 10$ in (a) and $N = 64$ in (b)

6. Conclusion

In this study, a new hybrid collocation method, combined by the third kind Chebyshev polynomials and block-pulse functions was proposed to solve higher-order initial value problems. Our method is capable of solving HOIVPs on large domains. HOIVPs on large domains are solved using other methods including exponentially fitted Runge-Kutta methods (Vanden *et al.*, 2000). These methods do not have good accuracy results. Another advantage of our proposed method is its high accuracy. Comparision of results show that the suggested method is more accurate than other methods.

References

Abdel-Halim Hassan, I.H. (2004). Differential transformation technique for solving higher-order initial value problems. Applied Mathematics and Computation, **154**(2):299-311.

Allison, C. (1970). The numerical solution of coupled differential equations arising from the schrodinger equation. Journal of Computational Physics. **6**(3):378-391.

Awoyemi, D.O. (2003). A P-Stable linear multistep method for solving general third order ordinary differential equations. International journal of Computer Mathemathics, **80**(8):985-991.

Awoyemi, D.O. & Idowu, O.M. (2005). A class hybrid collocation methods for third order ordinary differential equations. International journal of Computer Mathemathics, **82**:1287-1293.

Berstein, R.B., Dalgarno, A., Massey, H. & Percival, J.C. (1963). Thermal scattering of atoms by homonuclear diatomic molecules. Proceedings of the Royal society of London A, 274. **1359**:427-442.

Canuto, C., Hussaini, M.Y., Quarteroni, A. & Zang, T.A. (2006). Spectral Methods: Fundamentals in Single Domains, Scientific Computation. Springer-Berlin. Germany, Pp. **91**.

Guo, B. (2000). Jacobi approximations in certain Hilbert spaces and their applications to singular differential equations. Journal of Approximation Theory, **243**(2):373-408.

Guo, B.Y. & Wang, L.l. (2004). Jacobi approximations in non-uniformly Jacobiweighted Sobolev spaces. Journal of Approximation Theory, **128**(1):1-41.

Fehlberge, E. (1969). Low-order classical Runge-kutta formulas with stepsize control and their application to some heat transfer problems. NASA Technical Report, **14**: Pp. 315.

Hsiao, C.H. (2009). Hybrid function method for solving Fredholm and Volterra integral equations of the second kind. Journal of Computational and Applied Mathematics, **230**(1):59-68.

Jiang, Z.H. & Schaufelberger, W. (1991). Block pulse functions and their applications in control systems. Springer-Verlag Berlin Heidelberg NewYork, **37**. Pp. 3.

John C. Mason & David Handscomb, D. (2003). Chebyshev polynomials. CRC Press LLC, **43**. Pp. 68.

Karimi Dizicheh, A., Ismail, F., Tavassoli Kajani, M. & Maleki, M. (2013). A legendre Wavelet Spectral Collocation Method for Solving Oscillatory Initial Value Problems. Journal of Mathematics, **33**: Article ID 591-636. http://dx.doi.org/10.1155/2013/591636

Kayode, S.J. (2008). An efficient zerostable numerical method for fourth order differential equations. International Journal of Mathematics and Mathematical Sciences, **176**: Article ID 364021. http://dx.doi.org/10.1155/2008/364021

Kayode, S.J. & Awoyemi, D.O. (2010). A multiderivative collocation method for 5th order ordinary differential equations. Journal of Mathematics and Statistics, **6**(1):60-63.

Lakestani, M. & Dehghan, M. (2010). Numerical solution of Riccati equation using the cubic B-spline scaling functions and Chebyshev cardinal functions. Computer Physics Communications, **181**(5):957-966.

Marzban, H.R. & Razzaghi, M. (2003). Hybrid functions approach for linearly constrained quadratic optimal control problems. Applied Mathematical Modelling, **27**(6):471-485.

Maleknejad, K. & Tavassoli Kajani, M. (2003). Solving integro-differential equation by using hybrid Legendre and Block-Pulse functions. International Journal of Applied Mathematics, **11**(1):67-76.

Maleknejad, K., Basirat, B. & Hashemizadeh, E. (2011). Hybrid Legendre polynomials and Block-Pulse functions approach for nonlinear Volterra–Fredholm integro-differential equations. Computers and Mathematics with Applications, **61**(9):2821-2828.

Olabode, B.T. (2009). A six-step scheme for the solution of fourth order ordinary differential equations. The Pacific Journal of Science and Technology, **10**(1):143-148.

Tavassoli Kajani, M., Ghasemi Tabatabaei, F. & Maleki, M. (2012). Rational second kind Chebyshev approximation for solving some physical problems on semi-infinite intervals. Kuwait Journal of Science & Engineering, **39**:15-29.

Vanden Berghe, G., De Meyer, H., Van Daele, M. & Van Hecke, T. (2000). Exponentially Runge-Kutta methods. Journal of Computational and Appled Mathematics, **125**(2):107-115.

Waeleh, N., Majid, Z.A. & Ismail, F. (2011). A new algorithm for solving higher order IVPs of ODEs. Applied Mathematical Sciences, **5**(56):2795-2805.

Waeleh, N., Majid, Z.A., Ismail, F. & Suleiman, M. (2012). Numerical solution of higher order ordinary differential equations by direct block code. Journal of Mathematics and Statistics, **8**(1):77-81.

Yüzbaşi, Ş. (2011). A numerical approach for solving a class of the nonlinear Lane-Emden type equations arising in astrophysics. Mathematical Methods in The Applied Sciences, **34**(18):2218-2230.

Yüzbaşi, Ş. & Şahin, N. (2012). On the solutions of a class of nonlinear ordinary differential equations by the Bessel polynomials. Journal of Numerical Mathematics, **20**(1):55-79.

Yüzbaşi, Ş. (2012). A numerical approximation based on the Bessel functions of first kind for solutions of Riccati type differential-difference equations. Computers & Mathematics with Applications, **64**(6):1691-1705.

Yüzbaşi, Ş. (2013). Numerical solutions of fractional Riccati type differential equations by means of the Bernstein polynomials. Applied Mathematics and Computations, **219**(11):6328-6343.

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طريقة رصف هجينة تقوم على دمج حدوديات شيبيشيف من النوع الثالث ودوال قالب – نبضة لحل مسائل قيمة ابتدائية من مرتبات عليا

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خلاصة

الهدف من هذا البحث هو أن نقترح طريقة رصف جديدة لحل معادلات تفاضلية، خطية و غير خطية من مرتبات عليا و ذلك على فترة كبيرة جداً، كما نقوم بحل المعدلة التفاضلية على فترة صغيرة. وتقوم طريقتنا الجديدة على اتحاد هجين لحدوديات شيبيشيف من النوع الثالث ودوال قالب نبضة. وفي طريقتنا المقترحة نقوم بتقسيم الفترة الكبيرة إلى فترات جزئية أصغر، و في كل فترة جزئية تحول طريقة الرصف المعادلة التفاضلية إلى مجموعة من المعادلات الجبرية. وبحل هذا النظام نحصل على حل تقريبي للمعادلة التفاضلية على الفترة الجزئية. ومقدار الخطأ للحل التقريبي له حد أعلى، وأن هذا الحل يمكن إنقاصه بزيادة m و N . الطريقة المقترحة هي أكثر دقة من الطرق السابقة . وتقوم الأمثلة العددية بإيضاح فاعلية و كفاءة الطريقة المقترحة مقارنة بالطرق المتوفرة.