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Processing ambient noise using wavelet analysis tools: the Iraqi Seismological broadband Network Data at Iraqi Meteorological Organization and seismology (IMOS)

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Abstract

The purpose of this study is to reduce the noise of passive seismic signals recorded across the Iraqi Seismological Network Data at Iraqi Meteorological Organization and seismology (IMOS). The seismic records from six broadband stations of IMOS are largely affected by human activities in the populated cities of Iraq. We utilized several well-developed amplitude power spectrum and wavelet analysis techniques to improve signal-to-noise ratio. The results obtained from this comparison show that continuous wavelet transform can largely improve signal with minimal changes in the waveform shape of interest, even in presence of high noise levels. In addition, has overwhelmingly strong vitality and gains substantial development in the field of signal processing. Several mother wavelets were tested and Morlet and Coiflet selected as the optimum waves based on the signal characteristics and the noise reduction objective (maximizing SNR or reducing errors). Hence, the quality of the final seismic signal is crucial for the moment tensor analysis. However, traditional filters including discrete wavelet packet transform and 1-D wavelet analysis methods can be affective to reduce distortion in the signal.

Keywords: Ambient noise; continuous wavelet transform; denoising; IMOS; wavelet transform.

1. Introduction

Seismic signals are usually expressed in transient waveforms that radiate from natural, local, or man-made seismic sources. It can be utilized to identify the source, analyze source processes, and study the structure of the propagation medium (Bormann and Wielandt, 2013).

The expression seismic noise refers to unwanted components of earth's motion that their sources are mostly incomprehensible, and they alter their nature and position with the frequency band state (Yanovskaya, 2012).

Seismograms reflect the common effect of the seismic source and the pivotal origin of information about both earthquake and engineering seismology. Nevertheless, these records are constantly exposed to contaminate through the noise, which, must be removed before use in seismic applications (Bormann *et al*. 2009).

Seismic signals are influenced by man-made activities and sensors (Zheng et al., 2007). The seismic signals jumble with environmental noise, secondary noise, and the noise of the instrument itself, which leads to an error of earthquake estimation and wrong ruling. The instrumental noise has the weakest effect among the three kinds of noise on seismic signals (Li Ying et al., 2006). Reducing these interferences largely contributes to improving signal-to-noise ratio ((Mousavi et al., 2016; Mousavi and Langston, 2017; Yang *et al*. 2020). Multiresolution analysis such as wavelet transform has advantages over classical detection techniques, as it is less sensitive to noise (Rangasamy and Subramaniam, 2017).

Different processing techniques are available to improve data quality, regulate computational parameters, correct data, and offering inversion (Sokos and Zahradník, 2008; see: http://seismo.geology.upatras.gr/isola).

The main objective of this study is to process passive seismograms recorded across the Iraqi network and to present the best technique that may reduce ambient noise according to their results' reflection and their quality of data improvements. This paper proposed many denoising methods with the filtering and wavelet techniques. The procedure for these methods is based on spectrum analysis for pre-and post-processing data, and the signal-to-noise overlap will be initially determined using analysis software. Then, improved subsequent analyses. The quality of the final seismic signal is crucial for the moment tensor analysis. The results obtained from this study provided two characteristics: first, a unique base for quality control; second, optimize the methods of data analysis in the Iraqi Seismological Network Data at Iraqi Meteorological Organization and seismology (IMOS).

2. Study area

Previously, Iraq seismic Network (ISN) founded in 1976, and currently (IMOS), consisting of six broadband seismic station three-component namely Baghdad (BHD), Mosul (MSL), Kirkuk (IKRK), Rutba (RTB), Badra (IBDR), and Rafai (RAFI) (Figure 1). Digital recording system in each station consists of STS-2 Streckisen seismometers and Quanterra Q330 digitizer acquisition system. All seismic stations are located within cities; buildings have been added near them recently, although local noise sources are different from station to station, such as roads, population density, and daily and seasonal variations. Ambient noise is dominated by human activities, heavy automobile traffic, and there is machinery such as diesel generators and water pumps near some stations (such as the BHD station).

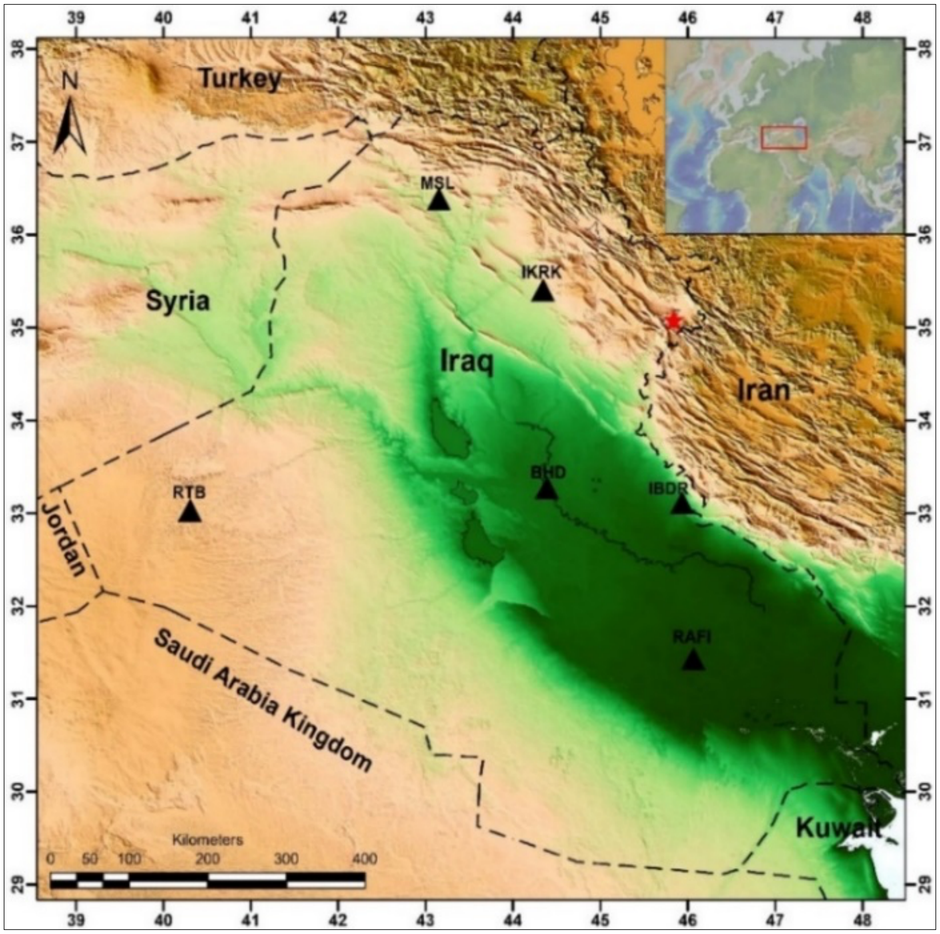

Fig. 1. Geographic distribution of seismic stations (triangles) and location of the event (star) 2017/12/11 dating 14:42:41 UTC, used in this study.

3. Materials and Methods

In the current study, a seismogram record from an earthquake located in the NE of Iraq is used. This earthquake happened at 11 Dec. 2017, 14:42:41 UTC time with a magnitude of 4.9 in Halabja-Sulaymaniyah area NE of Iraq. (Figure 1). The unfiltered vertical component (Z) of the aforementioned seismic event (which is recorded at BHD station) is shown in Figure (2a) using the ISOLA program. The spectral analysis of this component (Figure 2b) shows that the noise overlaps the main earthquake signal in the high frequency portion. This seismic signal was imported into MATLAB R2018a and taken as input data for decomposition using various methods of analysis such as wavelet analysis.

The analysis technique for separating noise from signal includes, digital filtering and methods based on wavelets techniques. Thus, demonstrates the benefits of utilizing wavelet method to increase seismic signal precision through improving the signal-to-noise ratio and reduce noise without distorting the original signal. From all processing sequence with wavelet-based transformations, it's clear that the proposed methods contribute as new opportunities for improving processing steps which leads at the end to more accurate results in the interpretation of seismological data. Conceptually, the continuous wavelet transform at the scale-translation level is better understood when compared to the short-time Fourier transform STFT. Nevertheless, seismograms whose frequency content vary with time, the STFT frequency transformation is not sufficient and suffer from the time- frequency resolution limitation as shown in all above figures. (Chakraborty and Okaya, 1995).

3.1. Discrete Wavelet Packet Transform (DWPT)

A wavelet is a waveform of efficiently finite duration. Lately, wavelet transform techniques have emerged and demonstrated potential in many applications as a powerful and effective signal processing tool in seismology (Chaudhry *et al*., 2007).

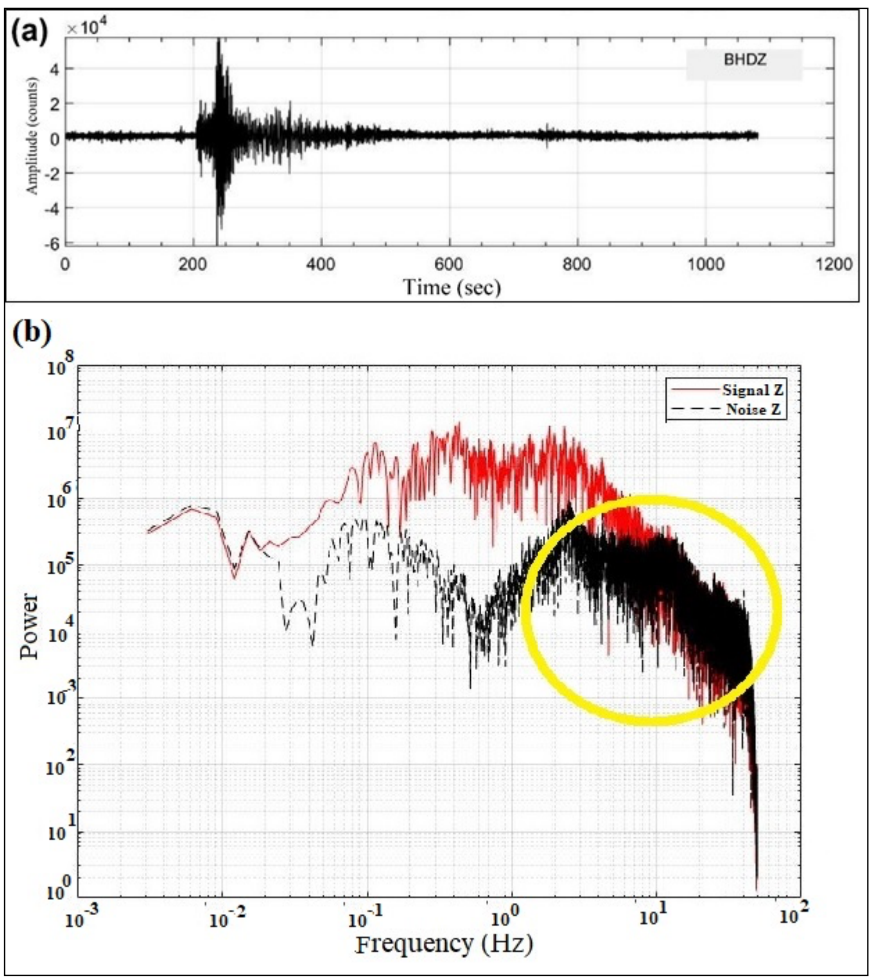

Fig. 2. (a) Z- component seismogram of 100 sample per second for a seismic event (original signal unfiltered) recorded at BHD station. (b) Spectral analysis of signal and noise (SNR), the graphs show the variation of noise as function of frequency where the noise clearly overlaps the main earthquake signal in the high frequency part (yellow circle).

Wavelets can be described as mathematical functions that divide data into various frequency ingredients in a way that allows them to study and compare each ingredient with precision separately according to its scale (Graps 1995). Seismic signals under natural conditions recorded by instruments can be represented by the following equation:

$$h(i) = c(i) + d(i) \quad (i = 1, 2, \dots N) \tag{1}$$

Where $c(i)$ express of ideal signal, $d(i)$ indicated doping noise, and $h(i)$ indicated the signals that were contaminated by noise.
Wavelet transform was used for the signal $h(i)$ in the next expression:

$$WT_{j,k} = \int h(i)\phi_{j,k}(i)di = \int c(i)\phi_{j,k}(i)di + \int d(i)\phi_{j,k}(i)di \tag{2}$$

where, ϕ named as essential wavelet or mother wavelet and j, k denoted discrete wavelet.

The DWPT wavelet have different cutoff frequencies and low and high pass filters to down sampling the signals at various gauges. Low pass filter supplies approximation-average coefficient of the signal distribution while high pass filter supplies detail coefficient for the same distribution (Polikar 1996). These coefficients have an important role and information as they assist to comprehend signal properties, as well as that the DWPT can available a time-scale realization of the digital signal utilizing digital filtering methods (Sripath 2003). Discrete wavelet decomposition in the detail section of the signal can be calculated from:

$$D_j(t) = \sum_{k \in z} C(j,k)\, \psi_{j,k}(t) \tag{3}$$

Where Dj is the detail portion, $C(j, k)$ is coefficients of the signal, ψ is the elected wavelet family and t is the time.

3.2. Continuous Wavelet Transform (CWT)

The CWT is used for mapping the changing properties of non-stationary signals. Thus, CWT is a time-frequency representation of a signal $f(t)$ that can be acquainted by the next equation:

$$W_f(a,b) = \int_{-\infty}^{\infty} f(t)\psi_{a,b}^{*}(t)dt = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} \psi_{a,b}^{*}\left(\frac{t-b}{a}\right) f(t)dt \tag{4}$$

where, $\psi_{a,b}^{*}$ is the mother wavelet coupling (Percival *et al.* 2004). The wavelet coefficients $W_f(a,b)$ are acquired by the constant changing of scale parameter and position parameter so as to determine the different parts of the signal and analyze various scale differences (Mallat 1999; Meyer and Ryan 1993). The constituent wavelets of the original signal are acquired by multiplying each coefficient by the suitable scaled and variable wavelets. The most used mother wavelet for the CWT is the "Morlet" function that extracts features with equal differences in time and frequency.

4. Results and Discussion

Comprehensive ambient noise processing methods are superior to individual processing methods in detecting pollution to the seismic signal through the noise and thus affect the results of improving the signal-to-noise ratio. These methods permit useful signal information to be extracted and extraneous information to be very effectively identified, isolated and eliminated. However, its accuracy and efficiency largely depend on the type of data, the nature of the station locations, and the type of method used for processing. In this study, we have applied more than one method and discussed each of them as follows:

4.1. Filtering technique

One of the most prevalent signal processing methods is signal filtering. Its purpose is to remove part of the signal in a specific frequency band and thus improve particular features and repress others. It is possible to use a variety of filters to improve certain stages of noise suppression, but perhaps can lead to new problems in the phase shifting (Bormann and Wielandt, 2013).

The basic assessment, of creating filtering is that beneficial information in signals is usually focused on the low frequency range while noisy signals are focused on the high frequency range (Cui and Wang, 2019). Hence, based on the frequency, effective signal and noise separation can be achieved.

Although there are many types of filters, in practical workable, Butterworth filters are used almost exclusively because they have a nice characteristic such that there is no resonance (or ripples) in the pass range and the angle frequency stays constant for any arrangement of a filter (Havskov and Ottemoller 2010). This particular filter greatly improved the signal-to-signal ratio (SNR) of the signal (Figures 3a and 3b), while at the same time it provides some distortion of the original signal and noise. Several researchers have developed optimized band-pass filters to reduce noise in seismic signals but have concluded that bandpass filtering introduces significant distortions to the signal (Douglas, 1997).

4.2.Denoising a signal using Discrete Wavelet Packet Transform (DWPT) technique

This type of wavelet has the advantage of transforming the signal at each of the frequencies; the time field and allowing for more effective signal analysis than the Fourier transform which is failed for analyzing the nonstationary signal while (DWPT) permits the components of a nonstationary signal to be analyzed (Sifuzzaman *et al*. 2009).

An appropriate choice is made for the analysis based on a new threshold function and an adaptive threshold was put forward according to the distortion problem of traditional threshold function denoising method. We have tested different numbers of scales for the wavelet transform to check the sensitivity of the method to the number of decomposition levels. So was selected the Coif wavelet as the mother wavelet, level 9, entropy threshold, and for the threshold parameter type balance sparsely-norm (see Misiti et al. 2013, for details). Once a random group of signals are decomposed by wavelet packets DWPT, the form of effective information and noise is significantly different in wavelet domain. Thus, it appears that DWPT technique has overwhelmingly strong vitality and gains substantial development in the field of signal processing.

Figure 4a shows decomposition Z-component seismogram obtained after applying a filter based on wavelet packets DWPT, wherein the signal contaminated with noise. Wavelet analysis has been used whereas the signal is decomposed in nine levels and the wavelet used is the Coiflet.

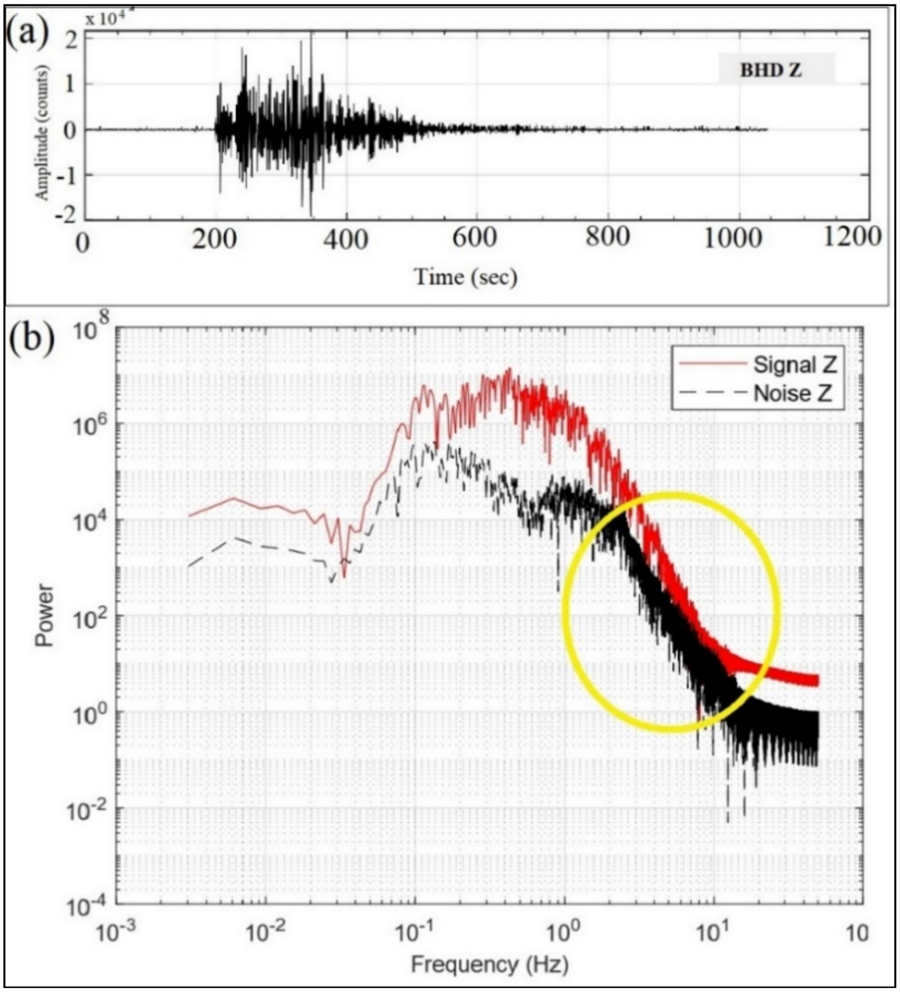

Fig. 3. (a) Shows seismogram filtered with a Butterworth filter.
Filters are band pass with corners 0.1 and 1 Hz (b) spectrum analysis of filtered signal.
Noise and signal are still overlapped (see the yellow circle).

It's clear figure 4b, how much enhancement in separating the noise from the signal (see yellow circle on the figure). Figure 4c shows the original coefficient of the seismic signal and its change with frequency (High-frequency increase upward). Figure 4d represents the cleaning of high frequency from the original seismic signal (Figure 4c). The figure 4 shows a very nice success method for denoising the raw seismic signal.

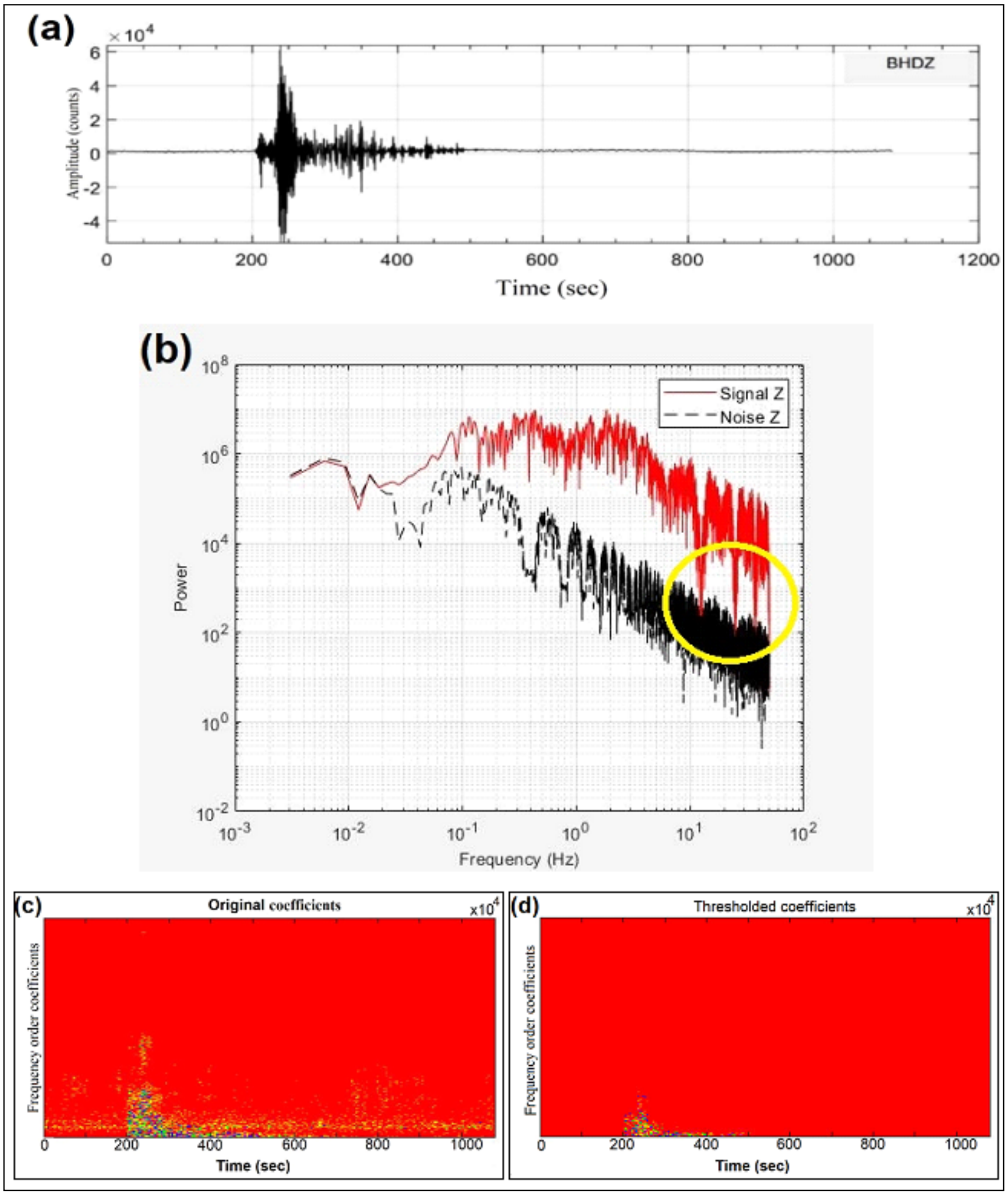

Fig. 4. (a) Seismogram filtered by the DWPT and a node-dependent threshold (b) spectrum analysis of filtered signal. (c) and (d) Time-frequency order coefficients decomposition using the DWPT for the original signal and a node-dependent threshold for the signal respectively. See the clean of (d) from high frequency data component which distributed randomly in (c) across time and frequency dimension.

4.3. 1-D Wavelet Analysis technique

Rounding approximation and details provide serviceable information in 1-D wavelet analysis for signal representation (Misiti *et al*. 2013). Utilizing wavelets to remove noise from a signal needs identifying which component or components contain the noise, and then reconstructing the signal without those components. After loading earthquake signal, we perform multi-level decomposition wavelet (by experimenting with many seismic events, we preferred to use 5 levels) by wavelet filters. Then constructed approximations (clean signal) as well as details (noise) from the coefficients

(Figure 5). However, a decomposed signal is reconstructed by taking a sum of approximation and five levels of detail coefficients. Then, perfect reconstruction error is calculated from a normalized disparity between the original signal and the reconstructed signal.

It is noted in figure 5, that consecutive approximations turn into less and less noisy as more and more high frequency information is filtered out of the signal. The level 5 approximation, a5, is perfectly clean as a comparison between it and the original signal (s).

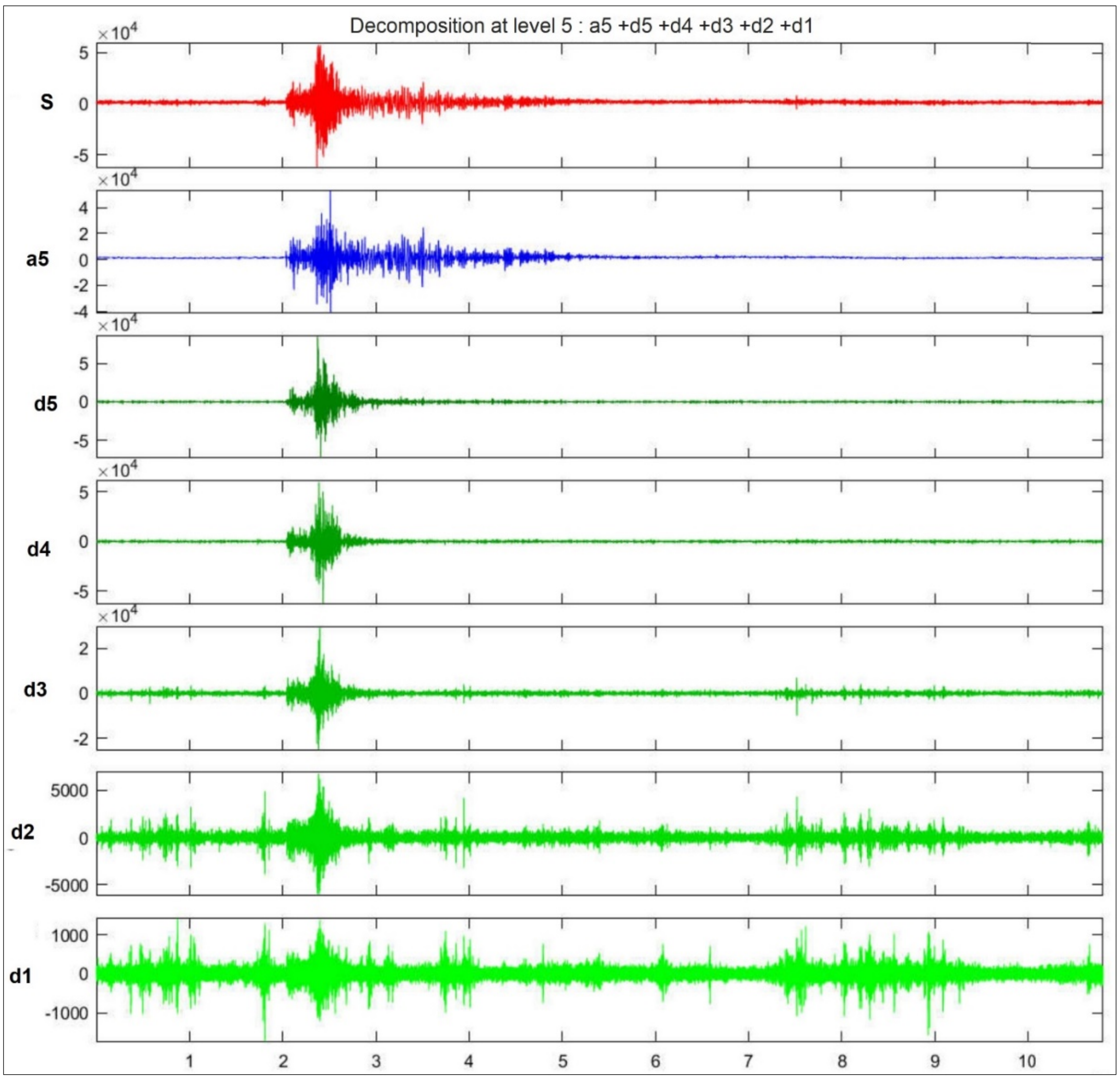

Fig. 5. Shows the approximation and detail of seismogram (s) filtered by decompose signal using 1D Wavelet Analysis Tool. Details signal from d1 to d5 represent five levels of detail (noise). The level 5 approximation, a5, is perfectly clean as a comparison between it and the original signal (s)

An illustration representation of average statistical outcomes for three various seismic signals are shown in figure 6. The original signal (Z-component) (red), residuals (noise) (yellow) and reconstructed (clean signal) (blue). Including measures of tendency (mean, mode, median), dispersion (range, standard deviation) and the tool provides frequency distribution diagram (histograms and cumulative histograms). The histogram shows a perfect typical unimodal curve (Figure 6c), compare this with figure 6a and b clearly declare the goodness of the analysis tools in separating noise.

Figure 7 shows decomposition Z- component seismogram obtained after applying a filter based on 1-D wavelet analysis. The results of this technique showed that after we passed a specific signal through the wavelet transform, the wavelet coefficient that representing useful information was closely related to each other between the different scales, but the wavelet coefficient representing the noise was not characterized by such character. We observe that this tool does significantly improve the SNR of the signal, while at the same time they introduce some distortion and overlap of the original signal.

4.4. Denoising a signal using Continuous Wavelet Transform (CWT) technique

Continuous wavelet transformation (CWT) has been applied to deal with the problem of the presence of ambient noise and its removal without affecting the transient seismic signal. The SAC (Seismic Analysis Code) format is used. CWT soft thresholding denoising is applied after the transform the original time series $f(t)$ into the CWT time-frequency domain so to get the CWT scalogram $W_f(a,b)$ (Figure 8). We attempted many of various mother wavelets and by comparing the RMS error between the input original signal and the final results; we achieve the minimum misfit utilizing the Morlet wavelet as the mother wavelet. The Morlet wavelet was because they give a CWT that can be explicated utilizing Fourier concepts where scale is directly associated to Fourier period. The Morlet wavelet also reduces the influences of seismogram Interruptions. The modulus of the CWT scalogram for the original signal is shown figure 8a, while figure 8b and c demonstrates that the original seismic signal appears unfiltered in SAC format and spectrogram analysis display shows the signal amplitudes as a colour intensity plotted as a function of the time-frequency domain respectively. The modulus of the CWT scalogram after soft thresholding and the final signal seismogram is shown in figure 9a. Whilst, it shows each of the figures 9b, c and d. Seismogram filtered based on CWT, spectrogram analysis of the time-frequency domain, and spectrum analysis of filtered signal respectively. The results showed that this method is one of the best ways to remove unwanted signals because it removes noise from the data very efficiently, maintains the characteristics of the original signal, has very strong vitality, and makes great progress in the field of signal processing. Deteriorated of seismic data often occurs during acquisition due to noise interference. Therefore, considered the first stage of seismic denoising is very important to the subsequent steps. There are clearly key parameters that require utmost precision in order to improve the accuracy of the results and it is very significant that only seismic and source characterization products utilize high-quality data. Nevertheless, spectral filtering is not efficacious for repressing noise that specifically includes the same frequency content as the signal, moreover; that it can distort the signal and change its features (Douglas 1997).

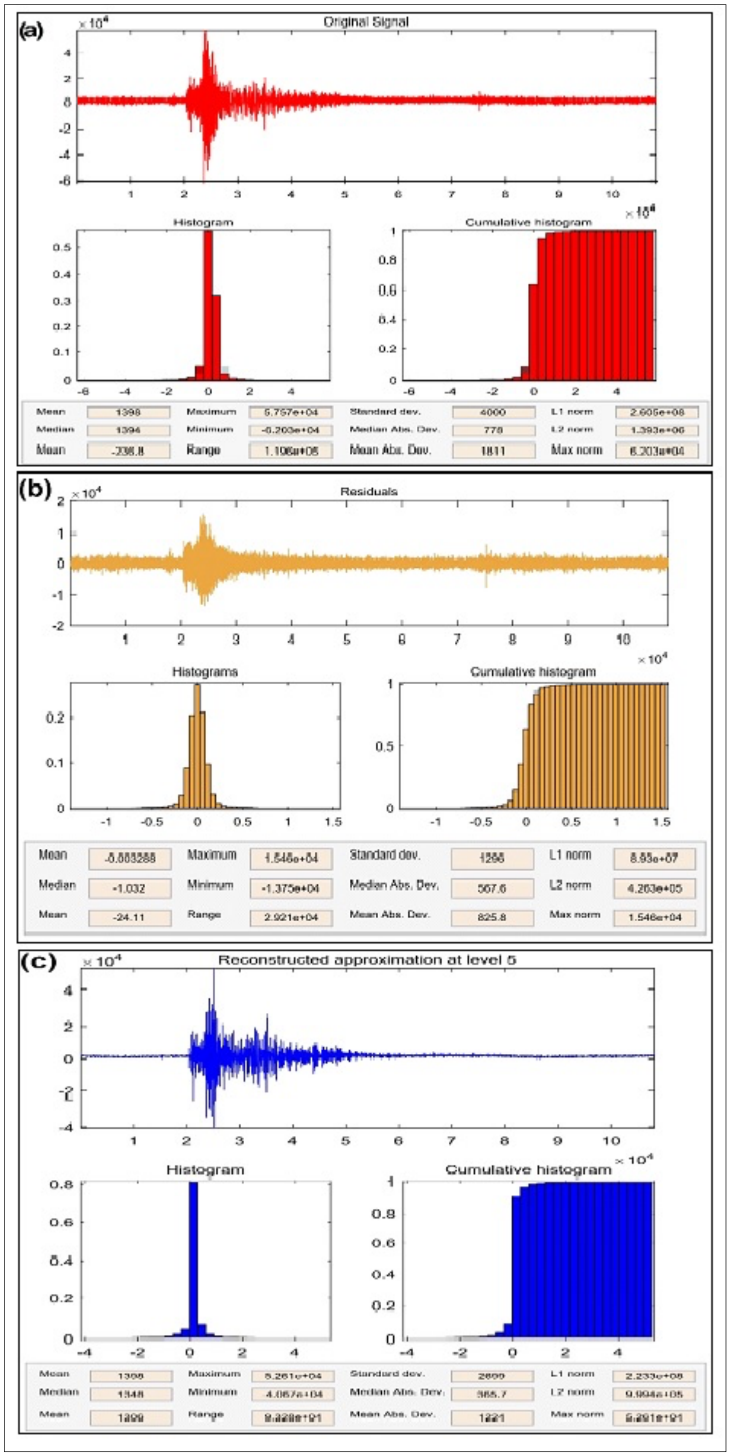

Fig. 6. (a) Displayed statistics of the original signal (Z-component)(red), (b) residuals (noise) (yellow) and (c) reconstructed (clean signal) (blue) from top to bottom respectively, and include tool provides frequency distribution diagram (histograms and cumulative histograms).

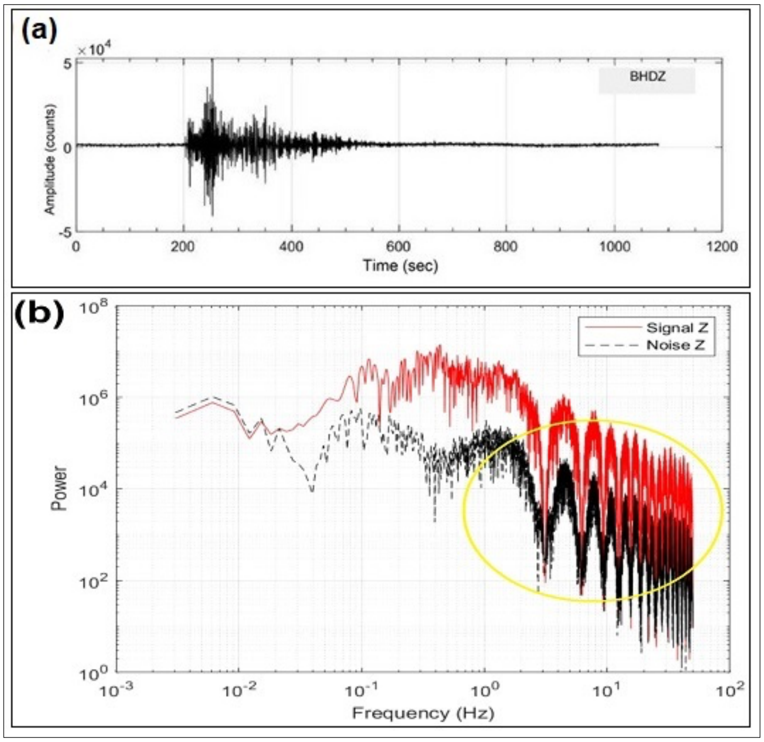

Fig. 7. (a) Seismogram filtered based on 1-D wavelet analysis. (b) Spectrum analysis of filtered signal. See the Yellow circle where some of interference between signal and noise exists.

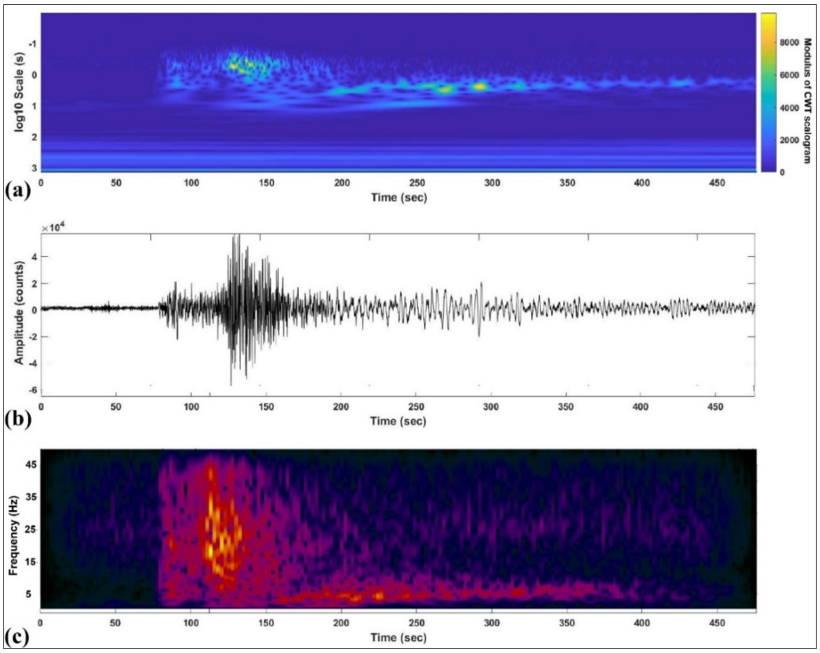

Fig. 8. (a) Shows the modulus of the Continuous Wavelet Transform (CWT) scalogram plotted in log10 (scale) vs. time for seismogram original signal unfiltered. (b) View the original seismic signal unfiltered appears in SAC format. (c) Spectrogram analysis display for original seismic shows the signal amplitudes as a colour intensity plotted as a function of the time-frequency domain.

Many new methods are achieved to reduce ambient noise from corrupted seismic records such as thresholding in the synchrosqueezed domain (Mousavi and Langston, 2017). But, the CWT method still and will continue for a long time as a dependable method for denoising seismic data. However, spectral filtering is ineffective to suppress noise, which includes precisely the same frequency content as the signal, moreover; it can distort the signal and change its features (Douglas 1997). The results of different processing steps provide different results. In Figures 2, 3, 4, and 7, the differences between the original data and the filtered data are compared which show the sections and the differences extracted between the original results and the processed.

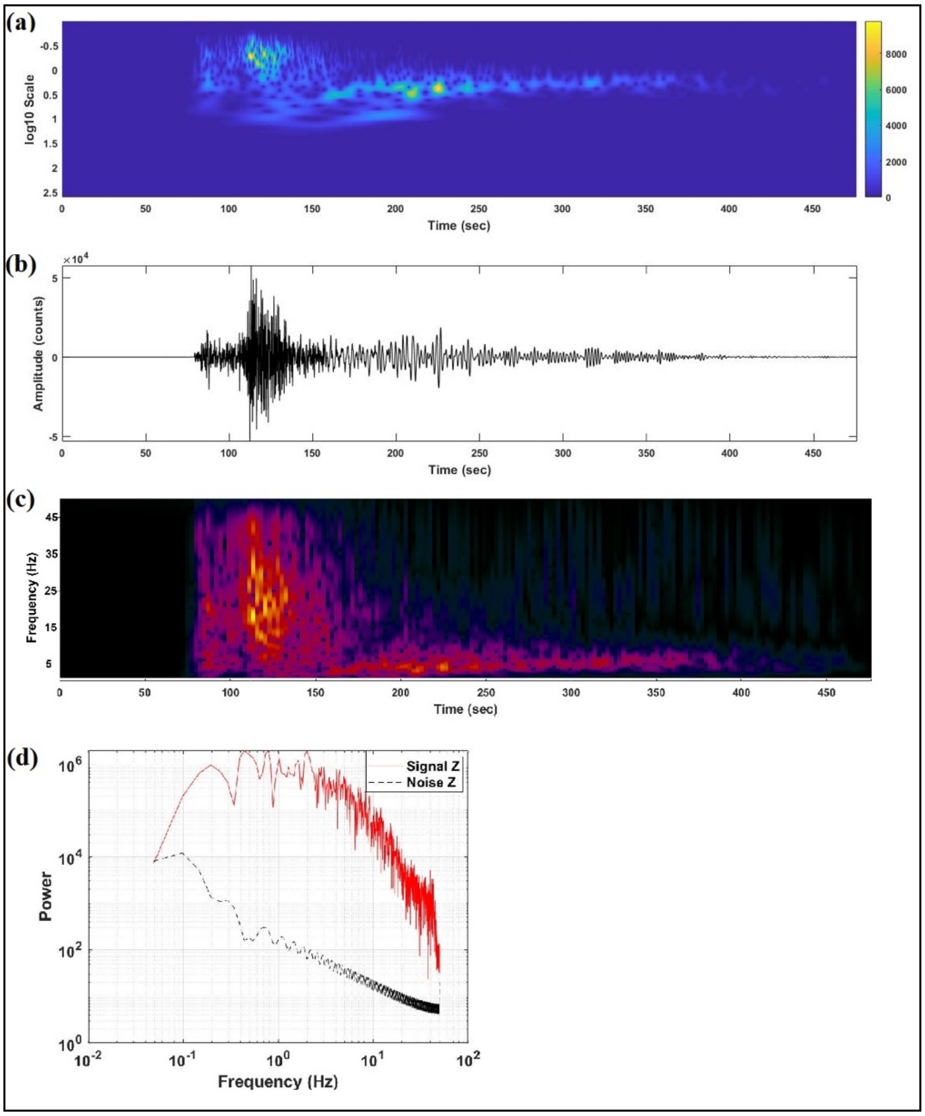

Fig. 9. (a) Shows the modulus of the Continuous Wavelet Transform (CWT) scalogram after soft thresholding and the final signal seismogram (b) Seismogram filtered based on CWT. (c) Spectrogram analysis display for final signal seismogram. (d) Spectrum analysis of filtered signal.

5. Conclusion

This paper focuses on four specific techniques for improving seismic signal data. A denoising style based on the wavelet packet transform is utilized to remove ambient noise from seismograms of Iraqi Network System (IMOS). The methods used have succeeded in removing random noise and improving the signal-to-noise ratio but to varying degrees. These methods showed that wavelets or wavelet packets improve considerably the signal-to-noise ratio compared to traditional filters. With filtering based on continuous wavelets, the resolution of seismic data was largely improved. This technique also reduced the distortion problem by using a more appropriate mother wavelet and decomposition level. Thus, achieved a resulting signal with a high SNR and without distortion in the signals. The (CWT) is distinguished from other ways by giving it information in both time, frequency ranges, and it is very beneficial for depicting non-stationary signals such as seismograms. Although other methods used in this field (traditional filters), (1-D Wavelet Analysis Tool) and (DWTP) can remove noise and improve seismic resolution, they are less effective than (CWT) with a minimal change in the waveform form of the original signal. Thus, the denoising CWT method performed better than other methods used in this research in terms of efficiency and accuracy since it removed the noise from the data quite efficiently. The filtering and 1-D Wavelet analysis techniques have greatly improved the signal-to-signal ratio (SNR) of the signal, while at the same time providing some distortion and interference of the seismic signal with noise.

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