Mathematical model of the influence of the time of start reaction in 100 m sprint run

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Abstract

The purpose of this study is to explain the influence of the time of start reaction t_R to the results of sprinters in a 100 m run, using a new simple mathematical model based on the measured values for distance *s,* corresponding time *t,* and t_R. The research is based on IAAF data obtained by measuring the segment length, the time of start reaction, transient times in 100 m run, and final times for the top sprinters C. Lewis (1988); M. Green (2011), and U. Bolt (2009) (men) and F. Griffith-Joyner (1988); E. Ashford (1988), and H. Drechsler (1988) (women). The values of the start reaction t_R for both male and female top sprinters indicate that there appear no substantial differences in the values of t_R based on gender which would directly favor male or female sprinters in achieving the top results in the 100m run. The influence of the time of start reaction t_R decreases exponentially with the time t during the run ($t > t_R$) and ends up at about 30 m, influencing the initial velocity v_R although it is not directly related to the result of the run. Due to its applicative simplicity, the presented mathematical model and related conclusions can represent a solid basis for future studies concerning sprint running.

Keywords: Model fit; speed-endurance; start reaction time; 100 m sprint.

1. Introduction

The mathematical modeling has become a powerful tool in almost every branch of basic or applied research, for example, BuHamra *et al.,* (2018); Goyal *et al.*, (2018); transport company maintenance operations, Savsar (2013) or mortality rate in Kuwait, Al-Jarallat (2010).

So, this is no wonder that it became important even in sports. We shall present a mathematical model describing the influence of the start reaction on the results of the sprinters, considering basic parameters. The time of start reaction (further on just "reaction time") t_R is an important factor for achieving the top results in the sprinter disciplines. This is primarily valid for top sprinters who try to gain a certain advantage during the first meters after the start and intend to keep it till the end of the race. Therefore, every sprinter intends to realize a functional relationship between the reaction time and start (initial) acceleration (a_R) to generate the result of the run.

To demonstrate the relationship and the influence of the reaction time to the initial velocity v_R, start acceleration (considered to be one of the most complex factors of sprint run), instantaneous and final velocity, many researchers (Andreacci *et al.,* 2002; Mero *et al.,* 1983; Moravec *et al.,* 1988; Coppenolle & Delecluse, 1989; Mero & Komi, 1990; Guissard *et al.,*1992; Schot & Knutzen, 1992; Harland & Steele, 1997; Pavlović *et al.,* 2014; Pavlović, 2015; McClements *et al.,* 1996) have tried to explain it in terms of the biomechanical analysis. The above authors using broadly the data on achieved results of the top sprinters, taken from the (IAAF, 2019) official reports treated by various standard statistical methods, offered the analyses showing that there exist certain relationships between the time of start reaction and the initial velocity and start acceleration which is important for top sprinters engaging maximally their psychophysical potentials during the run.

This can be further related to the results of the research of a group of authors (Susanaka *et al.,* 1998) who have indicated that corresponding individual capabilities (genetic, morphological, motoric, functional, etc.) of the sprinter together with the time of the start reaction time are good predictors of the sprint results. One should also mention here the work of the French group Morin *et al.* (2005, 2006), Rabita *et al.,* (2015), Samozino *et al.,* (2016), Slawinski *et al.*, (2015) who used very sophisticated measurement techniques combined with simple physical models and statistical treatment to obtain the relation between the velocity and the force (*F-v)* and from it deduce the kinematical and dynamic parameters.

Due to the importance of the initial velocity and start acceleration in the sprint disciplines, both based on the reaction time, we have initiated this research to use standard mathematical procedures to provide an analytical expression that unites two mentioned parameters, and which could be a model applicable to each sprinter. We hypothesize that in this way we could, without using statistical analysis, directly analyze and determine how the differences in the reaction time among male and female sprinters influence the initial velocity, start acceleration, maximal velocity, and the outcome of the race. The model will be tested on the data for 6 top sprinters.

We are aware of many recent studies by sophisticated methods like Piechota *et al.,* (2017) or Nagahara *et al.,* (2019), but we insist that the applicative simplicity of the mathematical model and presented results indicate that that the model can be used as a solid basis for further research related to sprinter running, both for men and women.

2. Materials and methods

2.1 Case studies

To demonstrate and test the application of our approach, we treated the measured data concerning the reaction time and segment times in the final 100-m runs for world-class sprinters–men: C. Lewis (Seoul Olympic Games, 1988); M. Green (Edmonton World Athletic Championship, 2001), and U. Bolt (Berlin World Athletic Championship, 2009), and women: F. Griffith-Joyner, E. Ashford and H. Drechsler (all three at the Seoul Olympic Games, 1988).

2.2 Theoretical Model

To develop mathematically a model that will explain the importance and influence of the reaction time to relevant parameters of 100m run, we shall assume that the sprinter moves along a straight line under the influence of the following forces: muscle force *F(t)* (Doder *et al.,* 2012), and force of the resistance F_v. Since our primary aim here will be to demonstrate how to reach the polynomial form for the velocity, following Keller (1975) we shall first assume that resistant force is proportional to the sprinter velocity–*kv*, with resistance coefficient *k.* Here, we follow closely our previous papers (Janjić *et al.,* 2014, 2017, 2019). The present work has been confirmed with the Ethical Code of the Research of the University of Novi Sad (2018-19-056).

Let us comment about the choice of the form *k v* for the resistance force of the air (medium) which appears due to the laminar flow of the air around the sprinter's body. There exists another form: $k_2 v^2$.

Probably the first research aimed to define an expression that would describe the effect of aerodynamic factors influencing the result of 100 m run were the equations of Hill. (1927) based on the measurement in the air (wind) tunnel. Later, these kinds of studies attracted more interest (Mureika, (1997, 2001); Davies, (1980); Ward-Smith, (1984, 1985, 1999); Dapena, Feltner, (1987); Pritchard, (1993); Linthorne, (1994, 1994a). They mostly treat the results of the influence of the medium (air) through the change of meteorological parameters of the atmosphere. We shall be interested in the results of Gŏmez *et al.,* (2013).

We write II Newton's law for the motion under the influence of the resulting force

$$F_{res} = F(t) - kv \tag{1}$$

leading to the linear differential equation (Keller, 1973)

$$m\frac{dv}{dt} = F(t) - kv \tag{2}$$

Introducing $\beta=k/m$ and assuming approximately constant segment force $F(t) = F \approx const$, we shall use the notation $F/\beta m = v_{ms}$ where $v_{ms}=\Delta s/\Delta t$ is the maximal segment velocity (Janjić *et al.,* 2014, 2017, 2019). Considering the following initial conditions considering the reaction time t_R

$$v = \begin{cases} 0 \; for \; 0 \leq t \leq t_R \\ v_R \; for \; t = t_R \end{cases} \tag{3}$$

the solution of equation (2) for the instantaneous velocity has the form

$$v = \begin{cases} 0 \; for \; 0 \leq t \leq t_R \\ v_R + (v_{ms} - v_R)\left[1 - e^{-\beta(t-t_R)}\right] for \; t \geq t_R \end{cases}$$

(4)

Distance *s* corresponding to the velocity (4) is

$$s = \begin{cases} 0 \; for \; 0 \le t \le t_R \\ v_R(t - t_R) + (v_{ms} - v_R)(t - t_R) - \dfrac{(v_{ms} - v_R)}{\beta} \end{cases}$$

$$\left[1 - e^{-\beta(t - t_R)}\right] for \; t \ge t_R \qquad (5)$$

The acceleration *a* is obtained by derivation:

$$a = \begin{cases} 0 \; for \; 0 \le t \le t_R \\ \beta(v_{ms} - v_R)e^{-\beta(t - t_R)} \; for \; t \ge tR \end{cases}$$

(6)

To determine the initial velocity v_R corresponding to $t = t_R$, based on data for *s, t,* and t_R, one must adapt first the expression (4). We wish to avoid saturation and produce an expression that could be valid during the whole run. We achieve this by expanding into Taylor's series the exponential term $e^{-\beta(t-t_R)}$in terms of $\beta\,(t - t_R)$ in the expression (4) for *v*, up to squares, so that the distance (equation 5) would be expanded up to third power, while the acceleration (6) would be only a linear function. The expansion leads to the following expressions for the instantaneous velocity *v,* distance *s*, and acceleration *a:*

$$v = v_R + \beta\,(v_{ms} - v_R)(t - t_R) - \tfrac{1}{2}\,\beta^2\,(v_{ms} - v_R)\,(t - t_R)^2 \; for \; t \ge t_R$$

(7)

$$s = \begin{cases} 0 \; for \; 0 \le t \le t_R \\ v_R(t - t_R) + \frac{1}{2}(v_{ms} - v_R)\beta(t - t_R)^2 - \frac{1}{6} \\ (v_{ms} - v_R)\,\beta^2(t - t_R)^3 \; for \; t \ge t_R \end{cases}$$

(8)

$$a = \begin{cases} 0 \; for \; 0 \le t \le t_R \\ \beta(v_{ms} - v_R)[1 - \beta\,(t - t_R)] \; for \; t \ge t_R \end{cases}$$

(9)

At this point let us compare our results with the results of Samozino *et al.,* (2015), and references therein. They assume that the force is proportional to v^2 and use this expression for calculating the resulting force. Yet, they assume the saturation type solution (4) in the acceleration phase, although it is the solution for the linear resistance force. We performed an expansion leading to the parabolic form of the velocity since it is then applicable for the complete run.

Another physical interpretation of the proposed approximation is the following: we assume that the acceleration instead of being constant, decays from the start acceleration as a linear function of time during the race so that the velocity reaches a maximum after which it decreases leading to the appearance of a negative cubic term in the expression for distance. This is Taylor's expression for acceleration (a) which is of the same form as the one used by Samozino *et al.,* (2015).

A simpler manner for writing (equation 8) is to introduce three positive coefficients P_1, P_2, P_3

$$s = P_1(t - t_R) + P_2\,(t - t_R)^2 - P_3(t - t_R)^3 \tag{10}$$

$$v = P_1 + 2\,P_2(t - t_R) - 3P_3\,(t - t_R)^2 \tag{11}$$

$$a = 2P_2 - 6P_3\,(t - t_R) \tag{12}$$

which can be fitted from the measured data for s, t, and t_R.

Comparing the equations (7) and (11) for $t=t_R$ it follows that initial velocity is $v(t=t_R)=v_R=P_1$. On the other hand start (initial) acceleration $a(t=t_R)=a_R=2P_2$.

Let us now discuss the meaning of this, initial velocity v_R. Its existence implies that now t_R the sprinters leave the blocks with this velocity. In our opinion, it is the consequence of coordination of all muscles enabling achieving this velocity practically as soon as they start moving. From the point of view of our model, the time in which the sprinters reach this velocity is negligible. Of course, this is a mathematical idealization, but one argument in favor of such an assumption is the fact that the model describes the motion well. As for the start (initial) acceleration, it is probably due to the extension of the front leg during the start.

After performing the fit of data to the third-order polynomial (eq.10) using programs Mathematica 7 and Origin 6.1, we can obtain an analytical expression for $s = f\,(t - t_R)$.

Now we go back and look at what would be the results if we assumed the resistance force to be quadratic ($F=k_2 v^2$). Generalizing the calculations of Gŏmez *et al.*,(2013)
We obtain the following expression for the velocity

$$v = \sqrt{\frac{F}{k_2}}\;\frac{v_R\sqrt{\frac{k_2}{F}} + tanh\left[\frac{\sqrt{Fk_2}}{m}(t - t_R)\right]}{1 + v_R\sqrt{\frac{k_2}{F}}\,tanh\left[\frac{\sqrt{Fk_2}}{m}(t - t_R)\right]} \tag{13}$$

Expanding it in terms of $(t - t_R)$ we arrive again at the expression (11) with the same coefficient signs. So, as far as we consider just the polynomial without interpreting the coefficients in terms of force and mass, the conclusions are still valid.

3. Results

We now present an example that can be used to test the applicability of the proposed mathematical model enabling the study of the reaction time t_R influence to the initial velocity, start acceleration, instantaneous and final velocity in achieving the top results. Our calculations are based on the data obtained by measuring the reaction times and segment times during the 100 m run of top sprinters: C. Lewis (1988); M. Green (2011), and U. Bolt (2009); F. Griffith – Joyner (1988); E. Ashford (1988) and H. Drechsler (1988). These values are listed in Table 1 for men and Table 2 for women.

Table 1. Transient and segment time, instantaneous velocities in the race at 100 m for C. Lewis 1988 at the Olympic Games; M. Green 2001, and U. Bolt 2009 at the World Championship

			C. Lewis 1998						M. Green 2001						U. Bolt 2009			
$s[m]$	$t[s]$	$\Delta t[s]$	$e^{-\beta(t-t_R)}$	$v_1[m/s]$	$v_2[m/s$	$v[m/s]$	$t[s]$	$\Delta t[s]$	$e^{-\beta(t-t_R)}$	$v_1[m/s]$	$v_2[m/s$	$v[m/s]$	$t[s]$	$\Delta t[s]$	$e^{-\beta(t-t_R)}$	$v_1[m/s]$	$v_2[m/s$	$v[m/s]$
0	0					v_0=3.45	0					v_0=3.61	0					v_0=3.42
		1.89						1.83						1.89				
10	1.89		$6.9\cdot10^{-2}$	$2.4\cdot10^{-1}$	4.92	5.16	1.83		$7.1\cdot10^{-2}$	$2.5\cdot10^{-1}$	5.08	5.33	1.89		$9.0\cdot10^{-2}$	$3.1\cdot10^{-1}$	4.81	5.12
		1.07						1.00						0.99				
20	2.96		$1.4\cdot10^{-2}$	$4.7\cdot10^{-2}$	9.22	9.26	2.83		$1.5\cdot10^{-2}$	$5.4\cdot10^{-2}$	9.85	10.39	2.88		$2.3\cdot10^{-2}$	$7.9\cdot10^{-2}$	9.87	9.94
		0.94						0.92						0.90				
30	3.90		$3.3\cdot10^{-3}$	$1.1\cdot10^{-2}$	10.60	10.61	3.75		$3.5\cdot10^{-3}$	$1.3\cdot10^{-2}$	10.85	10.85	3.78		$6.7\cdot10^{-3}$	$2.3\cdot10^{-2}$	11.04	11.06
		0.89						0.89						0.86				
40	4.79		$8.5\cdot10^{-4}$	$2.9\cdot10^{-3}$	11.23	11.23	4.64		$8.9\cdot10^{-4}$	$3.2\cdot10^{-3}$	11.23	11.23	4.64		$2.1\cdot10^{-3}$	$7.1\cdot10^{-3}$	11.60	11.61
		0.86						0.86						0.83				
50	5.65		$2.3\cdot10^{-4}$	$7.9\cdot10^{-4}$	11.63	11.63	5.50		$2.3\cdot10^{-4}$	$8.5\cdot10^{-4}$	11.63	11.63	5.47		$6.6\cdot10^{-4}$	$2.3\cdot10^{-3}$	12.04	12.04
		0.83						0.83						0.82				
60	6.48		$6.5\cdot10^{-5}$	$2.2\cdot10^{-4}$	12.05	12.05	6.33		$6.4\cdot10^{-5}$	$2.3\cdot10^{-4}$	12.05	12.05	6.29		$2.1\cdot10^{-4}$	$7.3\cdot10^{-4}$	12.39	12.19
		0.85						0.83						0.81				
70	7.33		$1.8\cdot10^{-5}$	$6.2\cdot10^{-5}$	11.76	11.76	7.16		$1.8\cdot10^{-5}$	$6.4\cdot10^{-5}$	12.05	12.05	7.10		$7.0\cdot10^{-5}$	$2.4\cdot10^{-4}$	12.34	12.34
		0.85						0.86						0.82				
80	8.18		$4.9\cdot10^{-6}$	$1.7\cdot10^{-5}$	11.76	11.76	8.02		$4.6\cdot10^{-6}$	$1.7\cdot10^{-5}$	11.63	11.63	7.92		$2.3\cdot10^{-5}$	$7.7\cdot10^{-5}$	12.19	12.19
		0.86						0.89						0.83				
90	9.04		$1.6\cdot10^{-6}$	$4.6\cdot10^{-6}$	11.63	11.63	8.91		$1.2\cdot10^{-6}$	$4.2\cdot10^{-6}$	11.23	11.23	8.75		$7.2\cdot10^{-6}$	$2.5\cdot10^{-5}$	12.05	12.05
		0.88						0.91						0.83				
100	9.92		$3.5\cdot10^{-7}$	$1.2\cdot10^{-6}$	11.36	11.36	9.82		$2.8\cdot10^{-7}$	$1.0\cdot10^{-6}$	10.99	10.99	9.58		$2.3\cdot10^{-6}$	$7.9\cdot10^{-6}$	12.05	12.05
t_R^*			0.136 [s]						0.132 [s]						0.146 [s]			
β			1.5186 [s^{-1}]						1.5571 [s^{-1}]						1.3757 [s^{-1}]			
m			81 [kg]						77 [kg]						86 [kg]			
Δs			10 [m]						10 [m]						10 [m]			
v_{max}			12.05 [m/s]						12.05 [m/s]						12.34 [m/s]			

t_R^*-reaction time is included into the time segment t from $0 - 10$ $[m]$

Using the measured values of s, t, and t_R from Tables 1 and 2, it is possible to perform a fit of distance s to a polynomial of the third-order (10) using the programs Mathematica 7 and Origin 6.1. From the fact that the coefficient P_1 equals the initial velocity v_R (according to eq. (11), we can conclude:

	Men
Lewis	v_{RL} =3.448 m/s
Green	v_{RG} =3.613 m/s
Bolt	v_{RB} =3.422 m/s

Women (14)

G. Joyner	v_{RJ} =3.604 m/s
Ashfrod	v_{RA} =3.637 m/s
Drechsler	v_{RD} =3.715 m/s

These values for the initial velocity v_R are listed in Tables 1 and 2 for $s = 0$ and $t = 0$. To analyze the influence of the time of start reaction to corresponding sprinter parameters, the expression (equation 4) for the instantaneous velocity for $t \geq t_R$ should be rewritten in the form:

$$v = v_R e^{-\beta(t-t_R)} + v_{ms}\left[1 - e^{-\beta(t-t_R)}\right] \; for \; t \; \geq t_R \quad (15)$$

Table 2. Transient and segment time, instantaneous velocities in the race at 100 m for F. Griffith-Joyner, E. Ashford, and H. Drechsler at the Olympic Games (1988)

Olympic Games (1988)																		
	Florence Griffith-Joyner						Evelyn Ashford						Heike Drechsler					
$s[m]$	$t[s]$	$\Delta t[s]$	$e^{-\beta(t-t_R)}$	$v_1[m/s]$	$v_2[m/s$	$v[m/s]$	$t[s]$	$\Delta t[s]$	$e^{-\beta(t-t_R)}$	$v_1[m/s]$	$v_2[m/s$	$v[m/s]$	$t[s]$	$\Delta t[s]$	$e^{-\beta(t-t_R)}$	$v_1[m/s]$	$v_2[m/s$	$v[m/s]$
0	0					v_0=3.61	0					v_0=3.64	0					v_0=3.72
		2.00						2.02						2.01				
10	2.00		$1.10 \cdot 10^{-1}$	$3.96 \cdot 10^{-1}$	4.45	4.85	2.02		$8.86 \cdot 10^{-2}$	$3.22 \cdot 10^{-1}$	4.51	4.83	2.01		$8.63 \cdot 10^{-2}$	$3.21 \cdot 10^{-1}$	4.54	4.86
		1.09						1.11						1.11				
20	3.09		$3.03 \cdot 10^{-2}$	$1.09 \cdot 10^{-1}$	8.89	9.00	3.13		$2.06 \cdot 10^{-2}$	$7.50 \cdot 10^{-2}$	8.82	8.90	3.12		$2.04 \cdot 10^{-2}$	$7.48 \cdot 10^{-2}$	8.83	8.90
		1.00						1.02						1.02				
30	4.09		$9.33 \cdot 10^{-3}$	$3.36 \cdot 10^{-2}$	9.91	9.94	4.15		$5.39 \cdot 10^{-3}$	$1.96 \cdot 10^{-2}$	9.75	9.77	4.14		$5.28 \cdot 10^{-3}$	$1.96 \cdot 10^{-2}$	9.75	9.77
		0.95						0.96						0.97				
40	5.04		$3.04 \cdot 10^{-3}$	$1.09 \cdot 10^{-2}$	10.49	10.50	5.11		$1.53 \cdot 10^{-3}$	$5.56 \cdot 10^{-3}$	10.40	10.40	5.11		$1.48 \cdot 10^{-3}$	$5.49 \cdot 10^{-3}$	10.29	10.29
		0.93						0.97						0.97				
50	5.97		$1.01 \cdot 10^{-3}$	$3.65 \cdot 10^{-3}$	10.74	10.74	6.07		$4.33 \cdot 10^{-4}$	$1.57 \cdot 10^{-3}$	10.30	10.30	6.08		$4.14 \cdot 10^{-4}$	$1.54 \cdot 10^{-3}$	10.30	10.30
		0.92						0.94						0.94				
60	6.89		$3.42 \cdot 10^{-4}$	$1.23 \cdot 10^{-3}$	10.87	10.87	7.01		$1.26 \cdot 10^{-4}$	$4.58 \cdot 10^{-4}$	10.63	10.63	7.02		$1.21 \cdot 10^{-4}$	$4.48 \cdot 10^{-4}$	10.64	10.64
		0.91						0.95						0.95				
70	7.80		$1.17 \cdot 10^{-4}$	$4.20 \cdot 10^{-4}$	10.99	10.99	7.96		$3.62 \cdot 10^{-5}$	$1.32 \cdot 10^{-4}$	10.53	10.53	7.97		$3.47 \cdot 10^{-5}$	$1.29 \cdot 10^{-4}$	10.53	10.53
		0.91						0.95						0.95				
80	8.71		$3.98 \cdot 10^{-5}$	$1.44 \cdot 10^{-4}$	10.99	10.99	8.91		$1.04 \cdot 10^{-5}$	$3.77 \cdot 10^{-5}$	10.53	10.53	8.92		$9.97 \cdot 10^{-6}$	$3.71 \cdot 10^{-5}$	10.53	10.53
		0.91						0.96						0.96				
90	9.62		$1.36 \cdot 10^{-5}$	$4.90 \cdot 10^{-5}$	10.99	10.99	9.87		$2.94 \cdot 10^{-6}$	$1.07 \cdot 10^{-5}$	10.42	10.42	9.88		$2.83 \cdot 10^{-6}$	$1.05 \cdot 10^{-5}$	10.42	10.42
		0.92						0.96						0.97				
100	10.54		$4.59 \cdot 10^{-6}$	$1.65 \cdot 10^{-5}$	10.87	10.87	10.83		$8.33 \cdot 10^{-7}$	$3.03 \cdot 10^{-6}$	10.42	10.42	10.85		$7.93 \cdot 10^{-7}$	$2.94 \cdot 10^{-6}$	10.31	10.31
t_R^*	0.131 [s]						0.176 [s]						0.143 [s]					
β	1.1808 [s^{-1}]						1.3139 [s^{-1}]						1.3120 [s^{-1}]					
m	59 [kg]						52 [kg]						61 [kg]					
Δs	10 [m]						10 [m]						10 [m]					
v_{max}	10.99 [m/s]						10.63 [m/s]						10.64 [m/s]					

t_R^*-reaction time is included into the time segment t from $0 - 10$ [m]

For practical reasons, it is suitable to denote the first term in (equation 14) with v_1, *i.e.* $v_1 = v_0 e^{-\beta(t-t_R)}$ and the second term with v_2 i.e. $v_{2=}v_{ms}\left[1 - e^{-\beta(t-t_R)}\right]$ so that eq. (15) becomes

$$v = v_1 + v_2 \quad (16)$$

Calculated values for v_1, v_2, and resulting velocity v are presented in Tables 1 and 2.

4. Discussion

Let us now analyze the above results in more detail. Equation (15) transparently shows that for $t = t_R$ $(s = 0)$, $v = v_R$, pointing to the maximal influence of t_R to the initial velocity v_R. The values (14) indicate that the largest influence on v_R, appears for the sprinter with the highest rate of the start reaction to which corresponds to the shortest reaction time t_R, in this case, it is Green (t_{RG} = 0.132 s) for man and G. Joyner (t_{RJ} = 0.131 s) for a woman.

On the other hand, higher values of t_R for male athletes Lewis and Bolt concerning Green, and Ashford and Drechsler concerning Joyner indicate that the reaction time influences a time delay of the sprinter at the very start, which is very difficult to compensate in the rest of the run 100-m (Smajlović & Kozić, 2006). Yet, this delay need not imply worse final placement, as demonstrated by Bolt (2009), who established a world record in 100-run (t = 9.58 s) even with the poor start time (t_{RB} = 0.146 s) and Ashford (1988) with the worst start time of all eight female sprinter finalists.

The results presented allow one to conclude that although the reaction time influences v_R, it is not directly correlated to the result (Tables 1 and 2). This fact speaks in favor of the researchers who claim that besides the start reaction time t_R which is one of the factors influencing the success (Dick, 1989; Pain & Hibbs, 2007), there is a more important factor influencing the result: initial velocity and start acceleration, achieved maximal velocity and speed-endurance (Martin & Buonchristiani, 1995).

The application of equation (15) allows the calculation of the values of instantaneous velocity vs time, which are listed in Tables 1 and 2 for each sprinter. Based on these results, one can analyze if there is any influence and of what kind of t_R to the start acceleration, instantaneous, maximal, and final velocity. We start with the analysis of the role of the coefficient $e^{-\beta(t-t_R)}$ For $t \geq t_R$, this coefficient governs the influence of time ($t > t_R$) in the equation (15) to velocity, and they are listed in Tables 1 and 2. Due to the increase of the time t during the run ($t > t_R$), an exponential decrease of this term, the observable influence of t_R ends at the distance of 30 m (according to Tables 1 and 2), i.e. the influence of exponential term over this distance is of order $10^{-4} - 10^{-7}$s so it is completely negligible.

The start acceleration which has the purpose to provide the sprinter with as high velocity as possible in the shortest time begins at the start and ends at about 30-m. Since this is the length corresponding to the end of the influence of reaction time, one can conclude that the start acceleration is based on the reaction time, as confirmed by the values of the coefficient $e^{-\beta(t-t_R)}$ listed in Tables 1 and 2 for s between 0 and 30-m. For this reason, in the segment corresponding to the length of the start acceleration, the sprinter must maximally exploit his sprinter capacity which manifests later in the segment of running with the maximal velocity. It is considered that the rate of the start reaction (shortest t_R) and start acceleration predominantly generate the result.

With as good as a possible value of these two factors, which is the result of 100-m run participation with even 64 % (Tellez & Doolittle,1984), the sprinter tries to achieve the maximal possible velocity in 50 – 70m. It is not always related to the result achieved (except for top results demanding velocity higher than 11 m/s) because, after 70m, the velocity decreases. In this last phase of the race, the dominant characteristic is speed-endurance whose role is to maintain as many percentages of the maximal velocity as possible till the end of the run.

5. Conclusions

The basic aim of this research was to use the standard mathematical procedure to provide an analytical expression as the model which can, in a simplified manner, using the measured values for *s, t*, and t_R, enable one to demonstrate the influence of the time of start reaction t_R onto initial velocity, start acceleration, maximal and final velocity in achieving the top results in 100-m run.

The research is based on (IAAF, 2019) data obtained by measuring the segment length, the time of start reaction, transient times in the 100-m run, and final times for the top sprinters C. Lewis (1988), M. Green (2011), and U. Bolt (2009) (men) and F. Griffith-Joyner (1988), E. Ashford (1988) and H. Drechsler (1988) (women).

Applying the new model to above mentioned (IAAF, 2019) data 100-m, we obtained the results which lead to several important conclusions. First, the values of the start reaction t_R for both male and female top sprinters indicate that there appear no substantial differences in the values of t_R based on gender which would directly favor male or female sprinters in achieving the top results. in the 100m run. Thus, we conclude that we deal with top sprinters who engage their mental and psychophysical capacities maximally.

The influence of the reaction time t_R decays exponentially with the time during the run ($t > t_R$) and ends up at about 30-m. The time of start reaction influences the start velocity v_R although it is not directly related to the result of the run. The start acceleration beginning at the start and ending at about 30 m, is based on the reaction time. After this distance, the influence of t_R is rather small, order of magnitude 10^{-4}-10^{-7} s so it is completely negligible and unimportant. The rate of the start reaction, corresponding to the short time $t_{R,}$ and the start acceleration generate to a high degree the maximal velocity (achieved at about 60–70m) and the final result although the maximal velocity is not always related to the final result of the run (Tables 1 and 2) since in the phase of the race after achieving v_{max}, the instantaneous velocity decreases so that further towards the finish, the dominant characteristic becomes the speed – endurance. It depends on the psycho-physical potentials of the sprinter during the run, as well as of his/her individual genetic, morphological, motoric, functional, and other capabilities, and it is not directly related to t_R.

Due to its applicative simplicity, the presented mathematical model and related conclusions can represent a solid basis for future studies concerning sprint running, both for male and female sprinters.

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