

## General modulation transfer function fitting equation for human eye with contact lens

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### Abstract

In this research, a general equation was derived to represent the modulation transfer function (MTF) of the human eye with the presence of the contact lenses (CL) and during the vision with monochrome or multi-wavelength (polychromatic) light. This equation was used to fit the MTF's curves for the eye with pupil diameters (EPD) (2.5, 4, 6, and 8 mm). The fitting equation accuracy was revealed by using the standard deviation (STD). The results indicated that the image MTF is sensitive to (EPD). Moreover, eye vision degrades as (EPD) increases due to the increased spherical and chromatic aberrations. The new equation consists of two exponential terms in addition to a correction polynomial function (H) to fit the EPD effect. The results showed that H function with power two is sufficient to give a good accuracy when the EPD is less than 4 mm, while H with power five is necessary when the EPD is larger or equal to 5 mm. The equation gives a high accuracy also in the case of vision with white light for the eye with a contact lens.

**Keywords:** Contact lenses; eye vision; fitting equation; image analysis; MTF.

### 1. Introduction

The human eye works as a complex refractive optical system consisting of the cornea, the anterior chamber, the crystalline lens, and finally the retina. This composition works together to form high images quality. Several models have predestined the human eye (Schulz, 2009), evaluated and analyzed the optical performance of eye and weak image boundaries (Almeida & Carvalho, 2007; Gharsallah & Braiek, 2017; Navarro *et al.*, 2006). The closest schematic eye model to the biological eye is Liou and Brennan's model (LBM). They suggested an aspherical gradient refractive index cornea and crystalline lens (Liou & Brennan, 1997; Liu *et al.*, 2005). Cataract surgeries (intraocular lens implementation), specular lens, and contact lens (CL) (Walsh & Charman, 1988; Watson, 2013) are used for vision-correcting (Bope & Kellerman, 2015).

Hybrid Diffractive/Refractive multi-focal intraocular lenses Implanted within human eyes were designed and evaluated by (Al-Hamdani, 2021). Material types selection used in optical elements fabricating is fundamental to obtain an ideal lens to correct the refractive errors. Early optical elements such as CL were made of glass; afterward, organic materials have been utilized to manufacture them (Efron, 2010; Walsh & Charman, 1988). Polymeric materials used in processing the vision correction elements are prescribed by several factors, such as flexibility of prepared film, transparency, gas and water permeability, diaphanous thermo-plastics, such as polysulfones (PSU), polyethersulfones, and polyphenylsulfones, preserve constant strength, hardness, and hydrolytic resistance and within a wide range

of variations in temperature values (Sastri, 2013).

## 2. Theory

The most important criterion is called modulation transfer function (MTF) in studying the optical system analysis and image quality. MTF is a criterion used to demonstrate eye vision or any optical system performance. The MTF is explained as the Fourier transform of the optical point spread function (PSF). MTF, at a specific frequency ( $\nu$ ) denoted by  $MTF(\nu)$ , is defined as the modulation of the image ( $M_i$ ) divided by the modulation of the object ( $M_o$ ), and is expressed as follows:

$$MTF(\nu) = \frac{M_i}{M_o} \quad (1)$$

For a perfect optical system, which is one with negligible aberration effects, the MTF is stated by:

$$MTF_{diffraction}(\nu) = \frac{2}{\pi} [\cos^{-1}(\omega) - (\omega)\sqrt{1-\omega^2}] \quad (2)$$

Where,

$\omega$  is the normalized spatial frequency =  $\frac{\nu}{\nu_0}$ ;  $\nu$  is the spatial frequency (cycle/mm),

$\nu_0 = \frac{1}{\lambda F\#}$  is the cut-off frequency (3)

$\lambda$  is the wavelength and  $F\#$  is the f-number.

Equation (2) is applied to an optical system with a circular aperture.

The calculation of the geometric MTF assumes a perfect optical system and judges the performance of a lens as a function of defocus. Diffraction effects are ignored. This assumption leads to (Haines & King, 1968; Shannon, 1994).

$$MTF_{geom}(\nu) = \frac{2J_1\left[\frac{\pi\nu\delta}{F\#}\right]}{\frac{\pi\nu\delta}{(F\#)}} \quad (4)$$

With  $J_1$  the first-order Bessel function and  $\delta$  is the defocusing error.

Human eye entrance pupil diameter (EPD) impact on the performance of the retinal image MTF for eyes without CLs and under

monochromatic light effect was studied (Al-Hamdani, 2018; Al-Hamdani, 2021). This article is based on Liou and Brennan's model (LBM) to design the human eye with a contact lens by using the zemax-premium 13 optical design program, (the design lens data editor was in Al-hamdani, 2020). The study was done in the presence of PSU-CLs and their influences on the MTF behavior for monochromatic 555 nm and polychromatic light (470, 510, 550, 610, 650) nm with weights of (0.091, 0.503, 0.503, and 0.107), respectively. An analytical fitting equation was evaluated. A general fitting equation for the MTF for the eye with and without CL, in white and monochromatic vision, was evaluated and analyzed. The accuracy of the fitting equation was studied by calculating the stranded error for each case.

## 3. Fitting formulas for the MTF

Many equations have been developed to represent the practical data, especially for the monochromatic light state (Bäumer, 2011). Initially, single functions were used, such as exponentials or Gaussians (Campbell & Gubisch, 1966; Dai, 2008; Flamant, 1955; Krueger & EA, 1973; Malacara *et al.*, 2004; Shannon & Wyant, 1992; Wetherell, 1980), which were not very precise to the experiment data. More recently, approximations with just two parameters (exponential functions) were submitted (Deeley *et al.*, 1991). These expressions provided a good fit at low and intermediate spatial frequencies but they were with a poor accuracy at high frequencies. By incorporating the EPD as a parameter in the expressions.

MTF would be described easily for any given pupil diameter. All of the preceding author's aims in this field were to introduce an accurate parametric model to describe the ocular MTF for every pupil size. The fitting for polychromatic MTF light is more complicated than monochromatic one, since the cut-off frequency depends on the wavelength, as shown in equation (3). In this work, the aim is to obtain a new general

analytical MTF equation ( by using Fortan 77), that can fit all the cases (monochromatic, polychromatic vision with and without CL).

#### 4. Results and discussions

The new general equation obtained in this work consists of two exponential functions and a correction polynomial function called error function (H) used only for high accurate fitting and for EPD larger than 5 mm. The new analytical expression allows calculating the MTF function as a function of spatial frequency and EPD for monochromatic and polychromatic light vision. This general analytical equation is expressed as:

$$MTF = Ge^{-B_0vd^2} + (1 - G)e^{-A_0vd^2} + H \tag{5}$$

$$F = (C_1 - C_2d^2) \tag{6}$$

$$H = Error\ function = p_0 + p_1v + p_2v^2 + p_3v^3 + p_4v^4 + \dots p_iv^i \tag{7}$$

Where  $d$  is the entrance pupil diameter (EPD):  $p_i$  is the polynomial parameters:  $A_0=0.0022$ ;  $B_0=0.00235$ ;  $C_1=-14.25$  and  $C_2=-0.13$  are constants. This equation was applied and analyzed in the following four optical cases:

Case 1: MTF equation for the eye without CL in monochromatic light vision.

The predictions of the general formula are shown along with the MTFs for each pupil diameter (2.5, 4, 6, and 8) for eyes without contact lenses in Figure 1. These fits are reasonably good with a given zero or low standard error for EPD 2 to 4 mm and the equation with a small number of terms (only three terms) is required. The fitting with five terms is necessary for EPD > 4 mm. Table 1 shows the values of the parameters in the correct function H in equation 7 and the low standard deviation between 0.0 and 0.004.

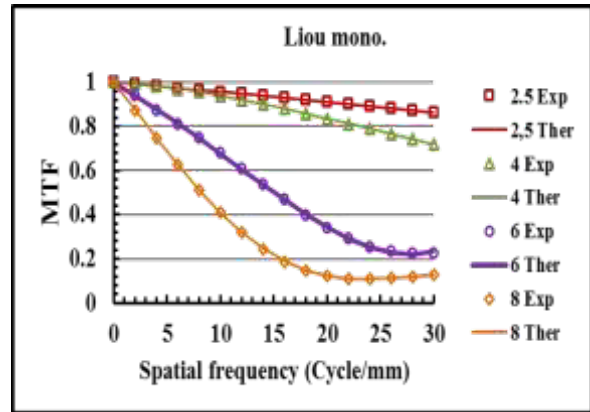


Fig. 1. Experimental (LBM) MTF (Exp) and the theoretical (Ther) from the fitting of equation (6) for the eye without contact lens in monochromatic light.

A similar Figures 2, 3, and 4 and Tables 2, 3, and 4 are illustrated for the following three cases respectively as follows:

Case 2: MTF equation for the eye without CL in polychromatic light vision .

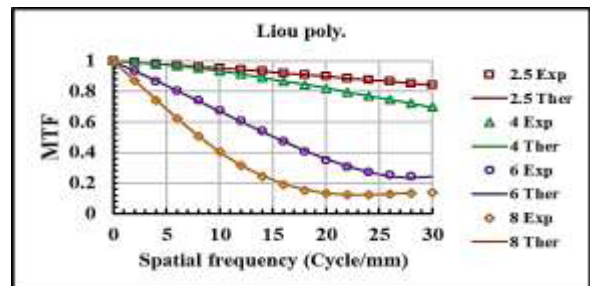


Fig. 2. Experimental (LBM) MTF (Exp) and the theoretical (Ther) from the fitting of equation (6) for the eye without contact lens in polychromatic light.

Case 3: MTF equation for the eye with CL in monochromatic light vision.

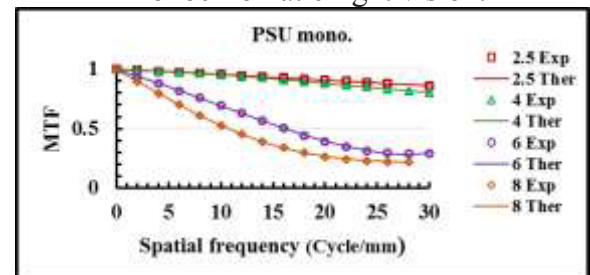
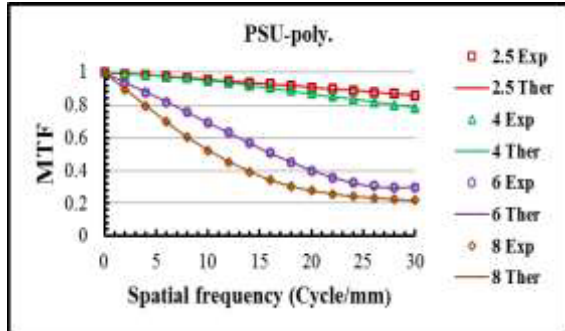


Fig. 3. Experimental (LBM) MTF (Exp) and the theoretical (Ther) from the fitting of equation (6) for the eye with CL in monochromatic light.

Case 4: MTF equation for the eye with CL in polychromatic light vision.



**Fig. 4.** Experimental (LBM) MTF (Exp) and the theoretical (Ther) from the fitting of equation (6) for the eye with CL in polychromatic light.

From all the four cases results in Figures 1-4 and Tables 1-4, one can observe that for a small EPD (0-4 mm), the fitting accuracy is reasonably well with only three terms of the correction function H (equation 7). While for EPD > 4 mm, the same fitting equation 6 gives an accurate fitting with 5 terms. The fitting equation used in this work consists of two exponential functions and a new correction function H to the generalized equation 6 for all pupil diameters in the moonlight and white light. The generalization of the equation to include white light is more complicated since the cut-off frequency depends on wavelength. The fit equation is proper for low, midrate, and high spatial frequencies.

**Table 1.** The P's parameters value and standard deviation for each EPD in monochromatic light without CL.

EPD mm	P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	ST Dev.
2.5	5.76E-04	2.90E-03	-4.44E-05			0
3	1.07E-03	9.89E-04	-5.21E-05			0
4	2.21E-03	-4.02E-03	4.87E-05			0.001
5	3.37E-03	-7.80E-03	2.63E-07	3.18E-05	-7.66E-07	0.003
6	4.02E-03	-1.11E-03	-1.96E-04	4.46E-05	-1.24E-06	0.004
7	8.37E-04	1.24E-02	8.52E-04	-1.18E-04	2.38E-06	0.003
8	2.98E-03	-2.66E-02	3.81E-03	-1.71E-04	2.35E-06	0.002

**Table 2.** The P's parameters value and standard deviation for each EPD in polychromatic light without CL.

EPD mm	P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	ST Dev.
2.5	3.69E-04	3.08E-03	-2.98E-05			2.5
3	7.81E-04	1.24E-03	-3.56E-05			3
4	1.14E-03	-3.25E-03	4.71E-05			4
5	4.33E-03	-7.78E-03	1.33E-04	2.34E-05	-6.54E-07	5
6	4.00E-03	1.20E-03	-4.33E-04	4.82E-05	-1.20E-06	6
7	-1.21E-03	1.61E-02	2.04E-04	-8.85E-05	1.99E-06	7
8	2.53E-03	-2.53E-02	3.76E-03	-1.77E-04	2.53E-06	8

**Table 3.** The P's parameters value and standard deviation for each EPD in monochromatic light with CL.

EPD mm	P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	ST Dev.
2.5	0.000576	0.002897	-4.4E-05			0
3	0.001545	0.000647	-7.4E-05			0.001
4	0.00401	-0.00506	-1.6E-05			0.002
5	0.002798	-0.00838	-0.00017	3.19E-05	-6.8E-07	0.002
6	0.003496	-0.0019	-0.00031	4.45E-05	-1.2E-06	0.004
7	0.001143	-0.00225	0.000418	-3.3E-05	5.5E-07	0.001
8	0.001883	-0.03727	0.003211	-0.00011	1.34E-06	0.001

**Table 4.** The P's parameters value and standard deviation for each EPD in polychromatic light with CL

EPD mm	P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	ST Dev.
2.5	6.30E-04	2.84E-03	-4.05E-05			0
3	1.42E-03	7.97E-04	-6.12E-05			0.001
4	3.45E-03	-4.62E-03	-8.96E-06			0.001
5	3.42E-03	-8.43E-03	-5.57E-05	2.56E-05	-5.98E-07	0.003
6	3.63E-03	-6.45E-04	-3.97E-04	4.25E-05	-1.05E-06	0.004
7	1.47E-04	5.97E-04	8.21E-05	-2.42E-05	5.08E-07	0.001
8	1.15E-03	-3.50E-02	3.01E-03	-1.11E-04	1.42E-06	0.001

#### 4. Conclusion

The influence of the EPD on the quality of the human eye image with contact lens (CL) is demonstrated. Results revealed that, with a EPD of 8 mm, increasing the opening of the pupil can reduce the image clarity and cause a lack of vision because the aberrations are largest. Aberrations are absent at the lens opening size radius of 2.5 mm. The use of CL substantially improves the vision quality. The degradation caused by pupil size is reduced, thereby improving the performance of the eye, especially for polychromatic vision. The fitting equation with EPD less than 4 mm gives a small STD by using only three terms from the correction polynomial H. Five terms are necessary for high accuracy with EPD > 4mm. In this work, the resulted equation with two exponential functions and a corrected function can be

used to determine the MTF for monochromatic and polychromatic eye vision for different pupil diameters with zero or small error.

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